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(VOLUME I)

SPACECRAFT ATTITUDE CONTROL FOR EXTENDED MISSIONS

Noah C. New
Major USMC

May 1963

degree of Doctor of Science

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B.A.E., Georgia Institute of Technology, Atlanta, Ga., 1949
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ABSTRACT

This thesis considers the problem of providing attitude control for a spacecraft engaged in an extended mission. As a basis for the choice of a suitable attitude control system the following requirements are applied.

Maximum reliability
Minimum ejection of mass
Minimum average power
Minimum system weight
Minimum peak power

An interplanetary mission of 400 days duration is adopted as a general guide for the problem, but most of the equations and comparisons are presented in parametric form. Extended missions imply that a momentum exchange type attitude control system be used to minimize ejection of fuel mass, and the thesis primarily considers only systems of this type. The thesis derives the equations of motion for a spacecraft equipped with eighteen different control systems. The control system chosen to best satisfy the five design requirements is a system consisting of four gyro-type controllers arranged in two pairs with each pair operating back-to-back to minimize control cross coupling torques. One pair of controllers provide roll torques, the other pair provides pitch torques, and all four controllers contribute yaw torques.

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contribute yaw torques.

The stability and control analysis considers operation of the spacecraft in three modes.

Zero Input Mode
Rate Control Mode
Position Control Mode

Each of the modes are evaluated for roll motion by assuming negligible interaxial coupling, and the analysis includes operation of the controller gimbal angles to large angles.

Thesis Supervisor: Wallace E. Vander Velde

Title: Associate Professor of
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CHAPTER 1

STATEMENT OF THE PROBLEM

1.1 Introduction

This thesis is concerned with the problem of controlling the attitude of a spacecraft during an extended mission. The perfection of large booster vehicles and spacecraft life-support systems will make it possible for explorers to embark on some of the greatest adventures of all history. The first of these adventures, that of exploring the moon and the nearby planets, appears likely within the decade if a reasonable extrapolation of progress in space technology is assumed. As a prerequisite to these explorations it is necessary to build a spacecraft that has a high probability of completing the mission. The vast distances involved in traveling to a nearby planet necessarily imply a long mission time between launch and recovery, and therefore that vehicle systems also must be reliable and self-sustaining over long periods of time. Salmirs [(1)] states that the most important single consideration in any space vehicle operation is its reliability. As with any new mode of transportation the early models of spacecraft will undoubtedly be crude compared with future space limousines; but even though they be crude, manned spacecraft must be designed with the safety of the crew and reliability of the systems taking precedence over all other factors.

Spacecraft attitude control influences a number of the operations and system capabilities of a space vehicle. Of primary importance is the guidance and navigation system because

(1) Numbers refer to list of references and underlined terms refer to Glossary of Terms in Appendix A. 5

it is probable that initial journeys to planets will be in spacecraft which have a thrust-impulse sufficient only for the bare mission plus a small reserve. The fact that propulsive fuel will be limited makes it mandatory that guidance and navigation procedures must be formulated around optimum trajectories. Navigation theories such as those developed by Battin[2] are based on the use of optical line-of-sight measurements which require at least some part of the spacecraft to be non-rotating in inertial space. Guidance and navigation systems may have star-trackers, telescopes, sextants, radar antennas, radio antennas, and an inertial guidance measurement unit all of which require some degree of attitude control. Having determined that the spacecraft is not precisely following a preplanned reference trajectory, an astronaut may elect to perform a mid-course guidance correction. Vehicle attitude stabilization is required for the mid-course phase both to align the spacecraft prior to the initiation of corrective thrust and during the time the thrust is applied. Of course, during the time the thrust is applied it is necessary to have some form of thrust-vector control so that the time average of the pitching and yawing moments generated by the thrust vector are zero. This thesis assumes that these time averages are zero and is thus primarily concerned with attitude control of the vehicle when thrust is not being applied, and the spacecraft is in free-fall trajectory.

The determination of a navigation fix on an interplanetary mission will generally require the measurement of at least three angles, and these will either be the angles between known references within the solar system or between known references within the solar system and distant stars. Using the principle of the marine sextant, an optical instrument known as a space sextant mounted on a stabilized platform can measure an angle between two reference bodies to an accuracy of 6 to 10 seconds of arc. In this scheme the attitude control system of the spacecraft need not have extreme pointing accuracy since separate

stabilization is provided to the optical sighting instrument. An interplanetary spacecraft designed for a Mars excursion may have a mass in excess of a million pounds according to Ehricke[3], and such a vehicle would have a large moment of inertia. Using such a large vehicle as a base to be stabilized to a limit cycle of the order of a second of arc, optical sightings could be made with a precision theodolite to an uncertainty of less than two seconds of arc.

Attitude control is required for a number of other missions and functions of the spacecraft besides alignment for navigation and velocity changes. Some of these are

1. Alignment of solar energy-collecting devices
2. Directing radio frequency antennas
3. Optical and television photography
4. Maintaining thermal balance of spacecraft
5. Scientific measurements of the space environment
6. Orbit control, rendezvous, and docking maneuvers of the spacecraft
7. Protection against micrometeorites
8. Minimize radiation hazards
9. Optimize storage of cryogenic materials

No attempt has been made to list the above requirements in any order of priority because this would vary with the over-all concepts of a mission and the design of the spacecraft. For example, although Wallner[24] says space radiation is isotropic, if it is determined that radiation hazards can be reduced by aligning the spacecraft such that the crew compartment is in the shadow of a long thin vehicle body, then this attitude may be preferred for long periods of time during the midcourse phase. Regardless of the extent of attitude required the above list of requirements indicates that attitude control of some form must be provided to the spacecraft continuously; or, if not continuously, for a sub-

stantial fraction of the mission time.

Spacecraft attitude control is defined as the operation of aligning a coordinate frame fixed in the vehicle with a reference coordinate frame in response to command inputs and/or isolation of the vehicle from disturbing torques. Spacecraft attitude control systems can be considered to be of several types-momentum exchange systems, momentum transfer systems, and external torquing systems which react with external pressures or fields. Hauessermann [4] as well as other writers have used different classifications, and of course, a particular control system may be composed of a combination momentum exchange system and a momentum transfer system. Also, one may differentiate between active attitude control systems and passive attitude control systems, as well as a control system which can be considered to be a semi-passive attitude control system. Burt [5] considers the passive and semi-passive attitude control systems in his paper on the attitude control of earth satellites. Satellites which are spin stabilized are considered to be semi-passively stabilized. Strictly speaking the passive and semi-passive systems are not control systems but are considered to be stabilization systems since they cannot respond to command inputs. This thesis is primarily concerned with active attitude control systems.

The momentum exchange systems derive control torques from internal controllers which undergo a time-rate-of-change of angular momentum, and since they do not expend fuel mass they are ideal for reorientation in response to command inputs or compensation of disturbing torques which vary about a zero mean. The momentum transfer systems must eject mass from the vehicle to derive a torque, and stabilization for extended missions may require prohibitively large amounts of control fuel. Unfortunately all of the torque disturbances acting on a spacecraft do not have a zero mean, and some form of momentum

transfer system may be necessary. Haeussermann[6] and Cannon[7] consider combination systems as do most of the papers considering a momentum exchange attitude control system. Thus a reasonable combination may be a momentum exchange control system for primary stabilization with a back-up momentum transfer system to control the vehicle during the erection following booster separation, during re-entry and to occasionally desaturate the momentum exchange system as a result of disturbing torques on the spacecraft which have a non-zero mean.

To minimize the requirements for desaturation of the momentum exchange system consideration must be given to make the external geometry of the spacecraft symmetric to the disturbances. These disturbances can be expected to be collisions with photons, collisions with solid matter in space, and interactions with electric and magnetic fields. It may be impossible to find a rest position for the spacecraft in which all of the external torques give a zero sum and still satisfy the mission requirements. However, certain broad assumptions can be made such as, the direction of the photons of solar energy can reasonably be assumed along the radials of the Sun, and therefore it is reasonable that the projection of the spacecraft towards the Sun should have a symmetric or balanced arrangement.

In the comparing of momentum exchange systems with momentum transfer systems the common denominator is the initial launch weight. It is true that momentum exchange systems consume electric power, but electric power is capable of being generated in space where the mass consumed by a momentum transfer type system is not readily capable of re-supply. In choosing a suitable momentum exchange attitude control system this thesis considers the weight and power required for various momentum exchange systems used to stabilize a space vehicle which is disturbed by a random torque with zero mean.

1.2 Statement of the Objectives

The introduction of the preceeding section has been a discussion of the general requirements of a spacecraft attitude control system suitable for extended missions. These requirements listed in the preferred order of priority assumed in this thesis are as follows.

Maximum reliability
Minimum ejection of mass
Minimum average power
Minimum system weight
Minimum peak power

It is recognized that these requirements are highly interrelated and perhaps are incompatible; nevertheless, the situation demands a practical solution. Although maximum reliability is listed above all of the other requirements, the second requirement of minimum ejection of mass requires that careful consideration be given to the momentum exchange type attitude control system. Although a number of references are available to evaluate specific type systems, none of the references appear to be in a form suitable for comparing various systems.

Roberson[8-11] treats the general problem of the satellite vehicle which is stabilized to a rotating reference frame. For the interplanetary spacecraft, alignment with respect to a non-rotating inertial reference frame is found to be more convenient. Special cases of vehicle stabilization using inertia reaction wheels have been published by many authors[7, 12, 13, 14, 15 and 16]. Special cases of vehicle stabilization by means of gyro torquers are also covered by a number of writers[17, 18, 19 and 20]. Virtually all of the above references use different systems of notation, and it is difficult to directly compare the equations. Kennedy[20], as well as Ogletree[21] and Amster[22], considers special configurations

of twin-gyro torquers; and these equations are considered to be very useful, but again they are special applications. Wells[52] considers a four gyro system with the angular momentum of each gyro along a bisector of a regular tetrahedron. Because of the lack of a suitable set of equations that would enable the engineer to consider a variety of different momentum exchange type control systems, and because of a suggestion by Mr. B. M. Hildebrandt of MIT/IL that "it would be useful to have a generalized set of control system equations that would enable a comparison of the various systems" the first objective of the thesis was established; namely to derive general equations to adequately represent an arbitrary spacecraft attitude control system using momentum exchange systems composed of rigid masses. The first objective is accomplished in the work of Chapters 2 and 3 with Appendices A through G. Consideration of the case of fluid controllers is separately treated in a less rigorous manner in a later chapter.

The second objective is to determine a reasonable description for the torque disturbances most likely to be encountered by the spacecraft in a space environment. This work is contained in Chapter 4.

The third objective of the thesis is to select a specific configuration of momentum exchange system that appears to best satisfy the five general requirements. This work is contained in Chapters 5 and 6.

The fourth objective is to apply the chosen system to the control of the spacecraft defined in the mission assumed in section 1.3 using both deterministic and statistical inputs. Chapter 7 contains the work of the fourth objective.

The fifth objective is to analyze the results and present conclusions that may be applicable to the choice of a spacecraft attitude control system of the future, and this objective is accomplished in the last chapter of the thesis.

1.3 Statement of an Assumed Spacecraft Mission

Although the space environment may have a time variation it is not expected that the environment for different trajectories will vary appreciably for interplanetary travel to the near planets of the solar system; therefore, the naming of a particular destination does not greatly influence the problem, because all interplanetary travel will require extended missions. It does appear that many factors favor Mars as a likely choice for a target planet following the exploration of the earth moon, and it is believed that no loss of generality will result if the planet Mars is specifically assigned in this mission. To embark on a manned excursion of the planets which requires a round trip time considerably in excess of one year presents many problems with life support systems that jeopardize the safety of the crew; therefore, it is considered unreasonable to choose a time substantially in excess of one year. Until such time as ultrahigh energy fuels are harnessed the time duration of flight is not expected to be less than about one year. A mission time considered to be in the state of the art of space propulsion is 400 days; therefore, let us arbitrarily assume that the mission time is 400 days. Reference 30 considers various trajectory parameters for probes and round-trip missions to Mars; a 365 day mission with no wait at Mars as well as a 465 day mission with 100 days delay at Mars are found to be optimum. Kirby[31] considers a 200 day one-way time, and Shartel[32] uses 171.5 days for a one-way time. Reference 30 further establishes that a 63 percent increase in the minimum velocity required for the optimum transfer to Mars of 11,560 feet per second will shorten the mission time 50%. With the rapid progress of recent years in powerplant performance the 400 day mission time is considered reasonable.

Any interplanetary space mission can be divided into various phases, and each phase may be further divided into subphases. To be specific, however, let us consider the follow-

ing phases as representative of a typical mission.

Phase A	Launch of Spacecraft subassemblies into an Earth Parking Orbit.
Phase B	Assembly of spacecraft.
Phase C	Injection into interplanetary orbit.
Phase D	Outbound Orbit.
Phase E	Entry into Target-Planetocentric Orbit.
Phase F	Reconnoiter of Target Planet.
Phase G	Injection into return orbit.
Phase H	Return Orbit.
Phase I	Entry into Earth Parking Orbit.
Phase J	Re-entry into Earth's Atmosphere to Surface.

This thesis is primarily concerned with phases D and H which will probably consume a large percentage of the total mission time. Since large amounts of fuel mass are required to make the velocity changes in the transition between the various phases the vehicle mass and moments of inertia will change significantly. To account for this change the spacecraft will be identified by its phase title, thus Spacecraft D will be the spacecraft having the physical properties associated with the outbound orbit, Phase D, and it may even be necessary to differentiate between vehicle mass and inertia changes during Phase D. If so, Spacecraft D1 will be the initial configuration immediately after the injection into the interplanetary orbit of Phase C, and Spacecraft D2 will be the configuration after the first velocity correction, if significant, and so forth. Spacecraft H will be the return orbit configuration of Phase H and it may have sequence configuration if such exists. As a final example, Spacecraft J will designate the re-entry vehicle, and this spacecraft may be a tiny fraction of Spacecraft A which is the original spacecraft prior to earth launch.

It is believed that all of the phases will require some

degree of spacecraft attitude stabilization. At this point however, it is difficult to say if phases C and G require more or less vehicle attitude control than do phases D and H. Phases E, F, I, and J require precision control of both spacecraft attitude and the velocity vector relative to the planet.

Before the equations for the torques produced by the attitude control systems investigated in this thesis can be applied to actual missions it is necessary to have some specifications on the performance required and some parameters which define the physical characteristics of the spacecraft. One must also have some knowledge of the expected torque disturbances on the spacecraft in a space environment. The torque disturbances are discussed in Chapter 4 whereas this chapter considers the physical characteristics of the spacecraft, the controllability requirements, and the stability requirements. Some authors consider the last two requirements as a single one; however, the terms do present different concepts to this writer as well as others such as Draper [23]. To the extent possible the presentation of the spacecraft response to disturbances is presented in parametric form; therefore, the numerical stability and controllability requirements and the physical characteristics listed in the following few pages are presented primarily to give the reader a general concept of the job that is to be done.

1.31 Physical Characteristics of the Spacecraft

The primary physical characteristics that must be known of a spacecraft assumed to be essentially a rigid body operating in an extra-atmospheric environment is its mass distribution and its external profiles. Other characteristics that are required can be listed as follows.

1. Thermal absorbing and reflecting characteristics of the exterior of the spacecraft
2. Detailed knowledge of the exterior shape of the spacecraft

3. The stiffness or flexibility of the spacecraft
4. Electrical conductivity of the exterior surface of the spacecraft
5. The residual and operating magnetic dipole moments displayed by the spacecraft
6. Detailed information on all fluids and the motions of the fluids within the spacecraft
7. Detailed information on factors concerning crew and their anticipated movements

Since the mission is of a long time duration the spacecraft will be subjected to a dangerous level of cosmic radiation[24], and it will be necessary to provide extensive shielding for the crew. The mass of shielding together with the large quantities of food, water and oxygen for the crew, and the fuel to make the necessary corrections and to accomplish the reconnaissance at the destination planet all indicate that spacecraft D will be massive. Therefore one may speculate that spacecraft D should be assumed as large as possible without being absurd. The capabilities of the class Nova vehicle or a larger one is visualized[25]. With this reasoning as a guide the following table presents the primary physical characteristics of the spacecraft.

TABLE 1.31

TABLE OF PHYSICAL CHARACTERISTICS OF THE SPACECRAFT

	Mass (2)	I_X (4)	$I_Y = I_Z$	External Characteristics
Spacecraft A (1)	—	—	—	(Details Not Specified) (3)
Spacecraft B	76	4000	242,000	
Spacecraft C	—	—	—	
Spacecraft D	20	490	31,000	
Spacecraft E	11	272	4,720	
Spacecraft F	10.4	255	4,450	
Spacecraft G	—	—	—	
Spacecraft H	4.14	75	415	
Spacecraft I	1.44	26	139	
Spacecraft J	0.4	8	8	

Notes

1. Subphases may also be required.

2. Units are kiloslugs for mass and kiloslugs ft^2 for moments of inertia. Mass distribution follows EHRICKE [51].

3. External characteristics will vary widely with design. In general, however, symmetry with respect to axis aligned toward the sun is desired.

4. Moments of inertia based on homogenous cylinder with average density of one slug per ft^3. Fineness ratio 10 for B and D, 5 for E and F, 3 for H and I, and one for Phase J.

1.32 Controllability Requirements

The controllability requirements of the spacecraft are a measure of its ability to follow command inputs. Thus the maximum time rates of change in roll, pitch, and yaw as well as the times required to achieve these maximum rates are important controllability requirements. The ability to hold a fixed attitude during the application of an external torque to the spacecraft may also be a measure of the controllability of the spacecraft, but this seems less well defined than the rates of roll, pitch, and yaw. Unfortunately very little information is available to set the specifications for the controllability requirements in space. It is reasonable to assume that the rates required will be small for a spacecraft in a peacetime role. Therefore, let us assume that the following requirements are specified for the interplanetary spacecraft.

TABLE 1. 32

TABLE OF CONTROLLABILITY REQUIREMENTS FOR SPACECRAFT

	MAXIMUM RATES (Milliradians/sec.)			RISE TIME (Seconds)		
	Roll	Pitch	Yaw	Roll	Pitch	Yaw
Spacecraft A	—	—	—	—	—	—
Spacecraft B	5	5	5	20	20	20
Spacecraft C	5	5	5	20	20	20
Spacecraft D	15	15	15	6	6	6
Spacecraft E	20	20	20	5	5	5
Spacecraft F	20	20	20	5	5	5
Spacecraft G	20	20	20	5	5	5
Spacecraft H	25	25	25	4	4	4
Spacecraft I	50	50	50	2	2	2
Spacecraft J	50	50	50	2	2	2

1.33 Stability Requirements

The stability requirements of the spacecraft are a measure of its ability to settle to a prescribed alignment within a prescribed settling time without exceeding specified limits following the application of disturbing torques, and to hold a specified alignment in the absence of external disturbances. As suggested earlier the stability and controllability of a spacecraft are distinctly different in concept;however, in a closed loop system one usually thinks of the overall operation as a stability problem. In this thesis the difference in stability and controllability is explained as follows.

The difficulty encountered by a spacecraft which operates in a nearly void space is that there are no natural force fields of sufficient strength to provide passive "pendulum type" stability. The exception possible is the torque attributed to the gradient of a gravity field which is always available in some small magnitude in the space environment; however, the torques involved are extremely small as will be shown in a later section. So, at best the spacecraft exhibits what may be termed neutral static stability such as does a perfectly smooth sphere which rests on a level surface normal to a gravitational field, and is therefore a second order system having zero damping and restoring moments. If such a sphere is given a slight motion by an applied impulsive force it would continue to move away from its initial position with no tendency to return to its initial position. If the sphere subsequently came to rest we would say that the damping of the system is positive. If the sphere continued to move at a uniform speed,then we would say that no damping exists in the system, or that the system is undamped. If the sphere accelerated in its initial direction of motion we would say that the system has negative damping. The nature of the spacecraft is that of one that is essentially undamped, and as stated before, the spacecraft does not possess an acceptable equilibrium point. Some systems can be made to be stable by changing

the physical characteristics of the system. For example, the smooth sphere can be made stable about a point on the plane by providing curvature to the plane so that the sphere, upon being disturbed from its equilibrium point, always rolls back to the equilibrium point, assuming the damping of the system is positive. The spacecraft however appears to be a system which cannot be stabilized in such a simple manner. Of course, it is feasible to provide damping to such a system, but damping provides little unless the system has some degree of static stability, i.e. possesses an acceptable equilibrium point. Therefore, the conclusion is that if a system cannot be passively stabilized about a point by natural means then it must be actively controlled about that reference point by a control system operating in a closed loop. Then it can be said that the spacecraft may be stabilized to any desired attitude by commanding the spacecraft to assume the attitude throughout a control system operating in a closed loop. Of course, in stabilizing a spacecraft by such an active controller, there is required some sort of sensing to establish the reference to which the spacecraft is aligned.

In the study of feedback control systems the concept of stability and control is described in a different manner, but the idea is precisely the same as described above. For example, Truxal on page 325 of reference 28 discusses a tandem single-loop system similar to Figure 1.33 which is simply a control system driving a vehicle in a closed loop. The vehicle represents an element of the system that is fixed and the transfer function of the vehicle cannot be substantially changed. The control system must provide the necessary stabilizing influence on the closed loop so that the ratio of output/input is optimized while the ratio of output/disturbance is minimized. The usual solution to satisfy both of these requirements is to make the transfer function of the control system such that it has a high gain and it places all of the closed loop poles in a location to give suitable damping characteristics.

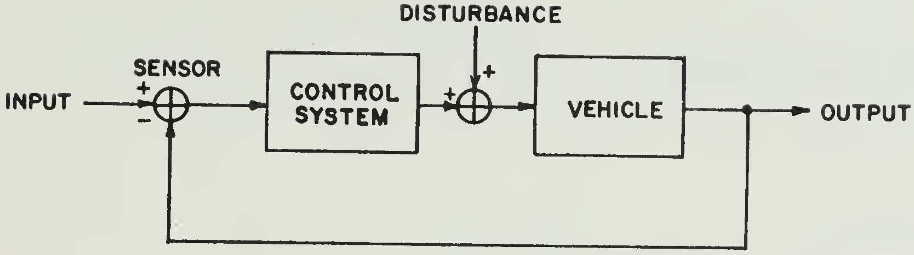


Figure 1.33 A simple feedback control loop

The discussion of stability and controllability as two separate requirements should not leave the impression that they are independently achieved. The stability and controllability of a spacecraft are closely related because they both are functions of the same feedback loop. Therefore the control engineer must find a proper balance between stability and control since excessive stability may preclude controlling the vehicle whereas excessive controllability may present the opposite problem where the vehicle is damaged by control actuation during a maneuver.

The stability requirements of a spacecraft vary widely depending on the particular task of the spacecraft. There follows in Table 1.33 a list of stability requirements which may be typical. However, again there is very little information available on these requirements, and so the figures must be taken lightly.

TABLE 1.33

TABLE OF STABILITY REQUIREMENTS
FOR SPACECRAFT

PRIMARY MISSION REQUIREMENTS	POINTING ACCURACY Roll	Pitch	Yaw	SETTLING TIME Roll	Pitch	Yaw
Spacecraft A	-	10	10			
B	100	100	100			
C	-	10	10		(Unknown)	
D (See note 1)	100	15	15			
Velocity Corrections	-	15	15			
Navigational Fix	100	5	5			
Radiation Shielding of Crew	-	-	-			
Communications	-	50	50			
Sun Pointing: Collector	-	40	40			
Sun Pointing: Solar Cells	-	10	10			
E	-	10	10			
F	-	15	15			
G	-	10	10			
H	-	10	10			
I	-	10	10			
J (L/D = 0.5)	-	75	100			
J (L/D = 2)	-	30	150			
SPECIAL MISSION REQUIREMENTS						
High Resolution Photography (Long Exposure)	0.0005	0.0005	0.0005		(Unknown)	
High Resolution Photography (Short Exposure)	1	1	1			
Television Weather Reconnaisance	10	10	10			
Telemetry Relay Transronder	20	20	20			
Optical Astronomy	0.001	0.001	0.001			
Radio Astronomy	1	1	1			

NOTES:

1. Each phase may contain subphases with different requirements.
2. Units are milliradians for pointing accuracy and seconds/milliradian error for settling time.
3. Pointing accuracy defined as maximum angular deviation allowed from the reference alignment.
4. Settling time defined at pointing error of two times pointing accuracy.

1.4 Summary

This chapter has been a statement of the problem of spacecraft attitude control for extended missions along with some of the reasons why attitude control is required. In this thesis the spacecraft attitude control system will be chosen on the basis of

Maximum reliability,
Minimum ejection of mass,
Minimum average power,
Minimum system weight, and
Minimum peak power.

Reliability is placed as most important with minimum ejection of fuel mass as second, and since momentum exchange type control systems require no expenditure of mass for torque disturbances having a zero mean value these systems appear attractive for extended missions. The thesis is primarily a study of momentum exchange type attitude control systems.

In practice the control system designer must have knowledge of the characteristics of the spacecraft, the controllability requirements, and the stability requirements. These assumed data have been given in Chapter 1 to give the reader a concept of the interplanetary mission, but in the subsequent chapters every effort will be made to present the equations in parametric form without the use of actual numerical data. Along with this, the equations will be solved analytically, if possible, and a computer will be used only where an accurate approximation can not be made analytically.

CHAPTER 2

DEVELOPMENT OF THE EQUATIONS OF MOTION

2.1 Introduction

The purpose of this chapter is to present the equations of motion for a spacecraft which is provided with a momentum exchange type attitude control system composed of a number of moving rigid components. Systems using fluid controllers are separately treated in a later section. The operation of attitude control is defined as the alignment of a vehicle-centered coordinate frame which is fixed in the vehicle with a reference coordinate system also centered in the vehicle. The equations are applicable to vehicles with either active or passive controllers. It is visualized that one mission of the spacecraft is an interplanetary mission during which the spacecraft will be in an orbit around the sun and remote from perturbing gravitational forces for long periods of time. During this time it is likely that one of the axes of the vehicle will be nearly aligned along the radial to the sun. The alignment of one axis of the vehicle with the radial line of the sun is defined as the sun-pointing mode and constitutes two-axis control. For other purposes requiring three-axis control, as well as more precision in the attitude holding operation, the alignment is defined as the precision mode.

For interplanetary travel in the foreseeable future the spacecraft will be in an elliptical orbit around the sun and in the sun-pointing mode it will have at least a small rotational velocity with respect to the stars of the order of 1^{o} per day. Because this rotation is small the reference frame chosen for the spacecraft attitude control investigation is a vehicle centered non-rotating inertial reference frame called simply the vehicle-centered inertial reference frame. In the close proximity of a planet the spacecraft may be referred to a planet centered

rotating reference frame which will require a coordinate transformation to the new frame.

In any theoretical development there must be a basic starting point, and for equations of motion this starting point is Newton's three AXIOMATA SIVE LEGES MOTUS.[26] From the second axiom is derived the principle of angular momentum which may be stated: With respect to an inertial frame of reference the time rate of change in angular momentum of any system is equal to the moment of force acting on the system. Thus if H is the angular momentum acting on the system and M_{ext} is the external moment applied to the space vehicle the principle of angular momentum can be expressed mathematically as:

$$\left.\frac{dH}{dt}\right]_I = \sum M_{ext}\Big]_I \qquad \text{(Eq 2.1.1)}$$

The problem of interest is the interplanetary space mission in which it is visualized that the spacecraft will be in a free fall orbit about the Sun. Because of the nature of space environment the attitude of the spacecraft has negligible effect on the six orbital parameters required to define the elliptical path about the sun so that these variables are independent of the variables defining the attitude of the spacecraft about its mass center. Therefore the task of providing spacecraft attitude control is that of controlling the roll, pitch, and yaw of the vehicle. The rates of roll, pitch, and yaw designated by p, q, and r thus represent three independent variables called the vehicle attitude rate variables. Suitable control moments provided by a spacecraft attitude control system must be available to the vehicle to provide attitude orientation to commands and to compensate for disturbing torques applied to the spacecraft. The momentum-exchange type spacecraft attitude control system must be a device which is capable of exerting torques on the vehicle by

causing a time rate of change in angular momentum of some mass elements integral to the control system.

The equations developed in this section postulate a particular type of controlling element which is a rigid gimballed rotor mounted in a case which has three degrees of freedom with respect to the spacecraft. Such a model can represent almost any type of controller and most systems can be constructed by superimposing the moment contributions from various controllers.

2.2 Symbols, Matrix Notation, and Coordinate Transformations

In the development of equations of motion of a spacecraft and its control system it is necessary to find expressions for the relative angular motion of the elements of the system with respect to each other and to an inertial reference frame. It is found that the equations are greatly simplified by the introduction of a clear and concise system of notation based on several sets of orthogonal Cartesian coordinates, and the use of matrix methods. The matrix methods follow closely those of reference 21, and the reader is referred to Appendix A of that reference for a complete explanation of the matrix notation used herein. Appendix A of this thesis is a brief statement of matrix notation together with a list of symbols peculiar to the equations contained herein.

The various coordinate reference frames are defined and pictorially sketched in Appendix B. All of these reference frames are othogonal Cartesian coordinate frames, and each frame is related to another by a 3 x 3 matrix called a coordinate transformation which are listed for reference in Appendix C. The relative angular velocities which appear frequently in the equations of motion become very complex when several coordinate frames are involved. For this reason, and again for reference, the relative angular velocities are listed in a separate section and can be found in Appendix D.

2.3 Statement of the General Equation

For spacecraft attitude control systems of the momentum exchange type composed of rigid moving parts a system model can be postulated to be a diametrically symmetrical rotor mounted in two orthogonal gimbals. Thus the postulated model is similar to a two-degree-of-freedom gyroscope, but to avoid confusion, since this gyroscope is used to provide control moments to the vehicle for the purpose of attitude control, the model is called an attitude controller or simply a controller. A spacecraft attitude system may be composed of one or more controllers. A controller of an active control system requires a torque generator and a signal generator for each degree-of-freedom.

Although the model of the controller is composed of a rotor and two gimbals, to avoid confusion in designating two gimbals the outside gimbal is considered to be the case of the gyro. Both two-degree-of-freedom and single-degree-of-freedom controllers are used as components of the control systems considered herein, and when single degree of freedom controllers are specified it is clear that the degree of freedom is the inner gimbal moving with respect to a fixed case. The postulated model is thus a rotor supported by a gimbal in a case, but the case acts like a second gimbal since it may have rotational freedom with respect to the speacecraft. See Fig. 2.3. The assumption of such a completely arbitrary element permits the treatment of any rigid body momentum exchange system. From the principle of angular momentum an equation for the total angular momentum of the spacecraft with respect to the Vehicle-Centered Principal Axis Frame, A, can be derived as follows. From equation 2.1.1

$$\left.\dot{H}_T\right]_I = \left.\sum M_{ext}\right]_I$$

Now $\left.H_T\right]_I = Q_{IA} \left.H_T\right]_A$ and $\left.\sum M_{ext}\right]_I = Q_{IA} \left.\sum M_{ext}\right]_A$

where Q_{IA} is coordinate transformation matrix given in Appendix C.

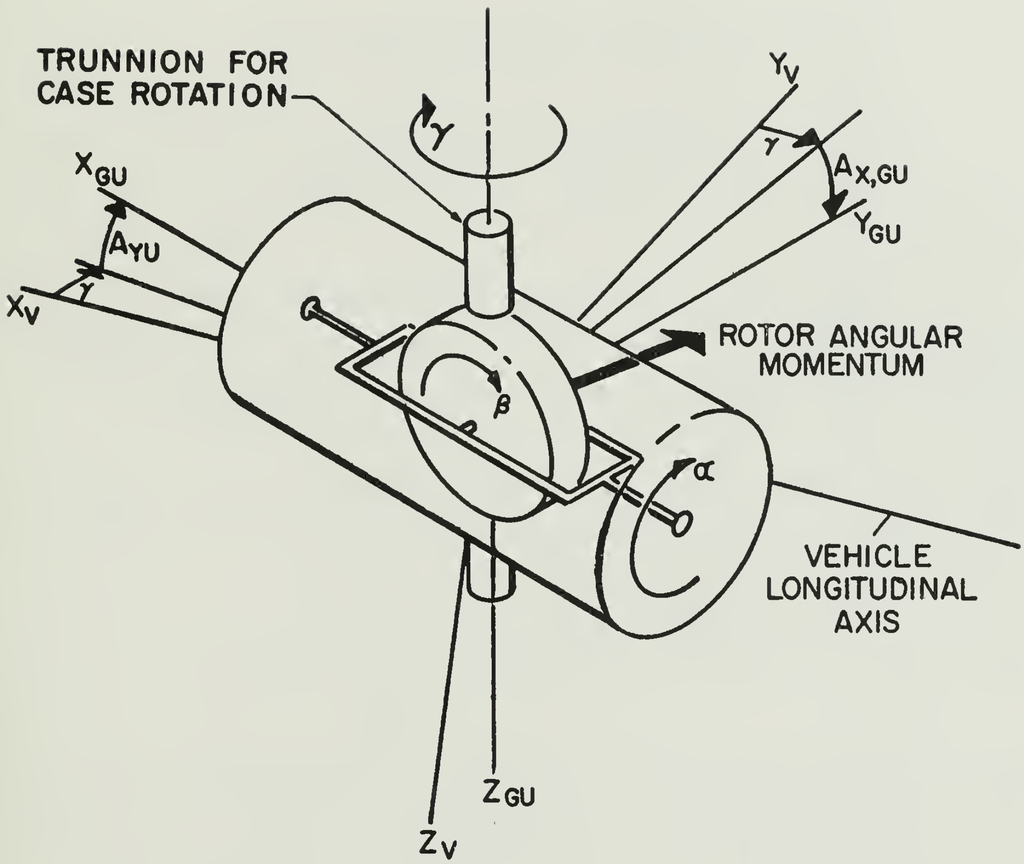


Figure 2. 3 An Illustration of the Model used in the development of the Control System Equations. Note that the case is located with respect to the vehicle by the angles γ, A_{YU} and $A_{X, GU}$. α is the gimbal position angle. β is the rotor position angle. See also Figures B. 9, B. 10 and B. 11 of Appendix B. Torque generators and signal generators are not shown but would be required for an active control system.

$$\text{Differentiating} \quad \dot{H}_T\Big]_I = Q_{IA}\,\dot{H}_T\Big]_A + \dot{Q}_{IA}\,H_T\Big]_A$$

$$= Q_{IA}\left[\dot{H}_T\Big]_A + W_{IA}\star H_T\Big]_A\right]$$

So that cancellation of Q_{IA} gives

$$\sum M_{ext}\Big]_A = \dot{H}_T\Big]_A + W_{IA}\star H_T\Big]_A$$

(Eq 2. 3. 1)

In the above equations a term like $W_{IA}\star$ has the definition given in Appendix A. 3. Thus if W_{IA} is an angular velocity with components given by a 3 x 1 column matrix $\begin{bmatrix} p \\ q \\ r \end{bmatrix}$ then $W_{IA}\star$ is defined as a 3 x 3 antisymmetric matrix

$$W_{IA}\star = \begin{bmatrix} o & -r & q \\ r & o & -p \\ -q & p & o \end{bmatrix}$$

For the postulated controller element the total angular momentum is the sum of the momenta of the vehicle, a case, a gimbal and a rotor for each of N controllers, and from motions of other internal masses.

$$H_T\Big]_A = H_{VF}\Big]_A + H_{VM}\Big]_A + \sum_{i=1}^{N}\left[Hc\Big]_A + Hg\Big]_A + Hr\Big]_A\right]_i$$

(Eq 2. 3. 2)

where subscripts VF refer to Vehicle Fixed Masses
and VM refer to Vehicle Moving Masses

also N represents number of controllers in control system.

If the expression for total angular momentum above is substituted into equation 2.3.1 and the terms for the moments contributed by the vehicle fixed parts, vehicle moving parts and controllers parts are kept separate the equation of motion for the vehicle can be written as

$$\sum M_{ext}\Big]_A = \sum M_{VF}\Big]_A + \sum M_{VM}\Big]_A + \sum M_{CS}\Big]_A \qquad \text{(Eq 2.3.3)}$$

where the vehicle fixed terms are

$$\sum M_{VF}\Big]_A = \dot{H}_{VF}\Big]_A + W_{IA} \star H_{VF}\Big]_A \qquad \text{(Eq 2.3.4)}$$

and the vehicle moving terms are

$$\sum M_{VM}\Big]_A = \dot{H}_{VM}\Big]_A + W_{IA} \star H_{VM}\Big]_A \qquad \text{(Eq 2.3.5)}$$

and the control system moment contribution $\sum M_{CS}\Big]_A$ is further resolved into case terms, gimbal terms and rotor terms

$$\sum M_{CS}\Big]_A = \sum_{i=1}^{N} \left(\left\{ Q_{A,GU} \left[\dot{H}c\Big]_{GU} + W_{A,GU} \star Hc\Big]_{GU} \right] + W_{IA} \star Q_{A,GU}\, Hc\Big]_{GU} \right. \qquad \text{(Eq 2.3.6)}$$

(Equation continued on next page.)

$$+ Q_{A,GIM}\left[\dot{Hg}\rfloor_{GIM} + W_{A,GIM} \star Hg\rfloor_{GIM}\right] + W_{IA} \star Q_{A,GIM}\, Hg\rfloor_{GIM} \quad \text{(gimbal terms)}$$

(Eq 2. 3. 7)

$$+ Q_{AR}\left[\dot{Hr}\rfloor_{R} + W_{AR} \star Hr\rfloor_{R}\right] + W_{IA} \star Q_{AR}\, Hr\rfloor_{R} \Big\} \quad \text{(rotor terms)}$$

(Eq 2. 3. 8)

Equation 2.3.3 with the definitions 2.3.4 through 2.3.8 is called the general equation of motion for a vehicle with a momentum exchange type control system composed of moving rigid components. The four parts of equation 2.3.3 are considered separately. First, the part representing the vehicle with its fixed parts will be evaluated. Then in the subsequent sections of this thesis the moments contributed by a momentum exchange control system consisting of several controllers will be derived. In later sections the external moments will be considered as well as the moments contributed by internal moving parts of the spacecraft of which crew movements can be expected to contribute a large part.

2.4 Determination of the Vehicle Contribution

Equation 2.3.3 has been divided into four separate parts to simplify the derivation of the various contributions. The term with subscript VF represents primarily the vehicle and all of the equipment that is substantially restrained to move with the spacecraft. It is clear that if a part of the vehicle has a movement then that movement will contribute to the angular momentum balance of the complete dynamic system and must be accounted for. It is for this reason that the vehicle has been separated into moving parts and fixed parts. From equation 2.3.4

$$\sum M_{VF}\rfloor_A = \dot{H}_{VF}\rfloor_A + W_{IA} \star H_{VF}\rfloor_A \qquad \text{(Eq 2.4.1)}$$

To simplify the equations as much as possible we consider vehicle attitude rate variables defined by equation D.2.2 of Appendix D.

$$W_{IA} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq 2.4.2)}$$

where p, q, and r are rates of roll, pitch and yaw of the spacecraft with respect to an inertial reference frame, frame I of Appendix D.

Now $H_{VF}\big]_A = I_A W_{IA}$ where $I_A = \begin{bmatrix} I_x & o & o \\ o & I_y & o \\ o & o & I_z \end{bmatrix}$

(Eq 2. 4. 3)

Thus $\dot{H}_{VF}\big]_A = I_A \dot{W}_{IA}$ if we assume I_A constant. (The moment of inertia of the vehicle may not be constant under certain thrusting operations, but this mode is covered under a later discussion). Substituting these expressions into equation 2.4.1 gives the moment contribution of the vehicle fixed parts to the general equation of motion for a spacecraft.

$$\sum M_{VF}\big]_A = \begin{bmatrix} I_x \dot{p} \\ I_y \dot{q} \\ I_z \dot{r} \end{bmatrix} + \begin{bmatrix} (I_z - I_y)\, q\, r \\ (I_x - I_z)\, p\, r \\ (I_y - I_x)\, q\, p \end{bmatrix} \qquad \text{(Eq 2. 4. 5)}$$

Equation 2.4.5 is to be substituted into equation 2.3.3 at a later point. The vehicle fixed parts give moments which are separated into two parts as shown by equation 2.4.5. The second part is called the contribution of the vehicle inertia cross coupling, and it is seen to vanish if the moments of inertia are all equal. For a long slender vehicle the moments of inertia are substantially different in magnitude and the inertia cross coupling cannot be neglected. A scheme for providing moments to neutralize the inertia coupling moments called compensation will be derived in section 3.6.

The moment contribution of the parts of the vehicle that move relative to the airframe are more difficult to define than the fixed parts. These moving parts can be classified as rigid parts, fluids and crew movements. The disturbances of the rigid parts can be determined by experimental and analytical methods during the component testing phase of construction of the spacecraft. The disturbances of fluids and crew are discussed in Chapter 4.

$$\sum M_{VM_A} = \begin{bmatrix} \text{All moving parts not} \\ \text{included in control} \\ \text{system} \end{bmatrix} + \begin{bmatrix} \text{fluids} \end{bmatrix} + \begin{bmatrix} \text{crew} \end{bmatrix} \quad \text{(Eq 2.4.6)}$$

2.5 Exact Equations for Case Terms, Gimbal Terms and Rotor Terms

The case, gimbal and rotor for a controller contribute moments to the spacecraft which are used to provide the required attitude control; therefore, in essence they are the most important terms. This section derives the exact expressions for these control moments for an arbitrary controller which fits the postulated model. Thus beginning with the case terms of the part of the equation represented by 2.3.6 which is

$$\text{case terms} = Q_{A,GU} \left[\dot{H}c\rfloor_{GU} + W_{A,GU} \star Hc\rfloor_{GU} \right] + W_{IA} \star Q_{A,GU}\, Hc\rfloor_{GU} \quad \text{(Eq 2.5.1)}$$

a straightforward expansion of the indicated matrices is accomplished. Appendix C contains the coordinate transformations, Appendix D contains the relative velocities required, and the case inertia is assumed to be

$$J_c = \begin{bmatrix} J_{cx} & o & o \\ o & J_{cy} & o \\ o & o & J_{cz} \end{bmatrix} \quad \text{(Eq 2.5.2)}$$

Carrying through the indicated operations gives the final expression for the case terms which is too lengthy for recording here and is given in Appendix E.

A completely analogous derivation of the gimbal terms is found from the part of equation 2.3.7 and for the rotor terms from 2.3.8. The results are found in Appendix E.

Inspection of the equations of this appendix shows that the expressions contain a large number of terms. However, in the consideration of an actual system many of the terms either vanish or can be neglected. For example, in a reaction wheel system the case and gimbal will be rigidly attached to the vehicle and thus the case and gimbal can be lumped with the spacevehicle. Further the rotor axis will be rigidly attached to the spacevehicle and thus many of the rotor terms vanish. In gyro type controllers the contributions of the moving case and gimbal can usually be neglected when compared to the angular momentum of the rotor. The exact equations contained in Appendix E must be studied for a particular system to determine which terms vanish or are small enough to be neglected.

2.6 Approximate Equations for a Single Controller

While the exact equations in the preceding section may be required in problems requiring extreme precision, the many uncertainties associated with a practical problem does not justify the inclusion of terms which are small when summed with terms several orders of magnitude larger. Therefore the equations can be simplified greatly if we assume that for gyroscopic type devices the angular momentum of the gimbals, case, and rotor about a diametral axis is small compared with the angular momentum of the rotor about its spin axis. See reference 27. Therefore, the case and gimbal terms can be neglected entirely, and only that angular momentum along the rotor element spin axis is retained. This can be stated mathematically as

$$Hr\rfloor_{GIM} = \begin{bmatrix} o \\ J_p\dot{\beta} \\ o \end{bmatrix} \qquad \text{(Eq 2.6.1)}$$

A single case, gimbal and spinning rotor is called a controller, and one can derive an expression for the moment contribution for a controller which has its spin axis aligned along the positive

y axis of the vehicle reference frame. A controller so aligned is defined as a number one controller. Appendix F contains equations for an arbitrary controller as well as a number of particular controllers which are numbered one through ten. These ten controllers are defined as representing reasonable components of possible complete control systems, and they will be used as building blocks to construct particular systems. If other orientations of the controllers are desired it is a simple matter to substitute the appropriate angles of the case into one of the ten defined positions or the arbitrary equation F. 1. 6 to arrive at the new expression for the moment contribution of the controller.

Also included in Appendix F. 2 are equations for inertia reaction wheels which are exact, but they are placed in Appendix F rather than Appendix E because the equations are restricted to wheels and cannot be applied to gyro type controllers. Certain terms are dropped from the exact equation to form the approximate equation given by equation F. 2. 8.

2.7 Summary

This chapter has presented the equation of motion of a spacecraft equipped with a control system composed of rigid rotating parts which are commanded to change their angular momentum and thus apply control torques to the spacecraft. The equations are based on a model consisting of a rotor mounted in a gimbal and held in a case. The rotor speed, gimbal angle, and case attitude relative to the spacecraft are all controllable in the general case. One or more of these controllers may be combined to form a complete attitude control system. The formulation of equations for complete systems is the subject of Chapter 3.

CHAPTER 3

EQUATIONS FOR COMPLETE MOMENTUM EXCHANGE ATTITUDE CONTROL SYSTEMS

3.1 Introduction

In the previous chapter there have been derived equations for various inertia wheel and gyroscopic type devices called controllers which are capable of generating control moments sufficient to accomplish the function of attitude control of spacecraft. In the present chapter it is desired to arrange several of these controllers together to form a number of different momentum exchange attitude control systems. Clearly all that is required to derive the moment contribution of a complete system is to select the controllers desired in the system and to sum their contributions.

After the moment contributions of several controllers are summed it is often necessary to derive a control logic that will reduce the number of degrees of freedom of the control system to three, or to diagonalize certain matrices in an effort to arrive at non-interacting control. This is accomplished by a control logic matrix transformation. Having reduced the equations to three control variables the equations are examined carefully to determine their independence, and a considerable portion of the chapter is devoted to a discussion of the isolation of the pitch, roll, and yaw control loops. This chapter is somewhat complicated by the introduction of several new definitions. If the reader desires a brief explanation of the definitions, refer to section 3.7 which presents the development in five steps.

The scheme used to isolate the control loops is called compensation, and it is found that three kinds of compensation can be provided. These are gyroscopic coupling, cross control

coupling, and spacecraft inertia cross coupling compensation. Figure 1.33 shows a simple feedback control loop with a sensor, a control system, and a vehicle indicated. The problem of the spacecraft is that of three of these loops for roll, pitch, and yaw, and the loops are interacting because of the coupling terms. The theory of this chapter provides non-interacting control by deriving a compensation which tends to eliminate the coupling between the loops. Thus the control system block of Figure 1.33 may be visualized as containing the compensation networks, the logic networks, and the controller dynamics. Figure 3.1 is a schematic of a control system showing these parts.

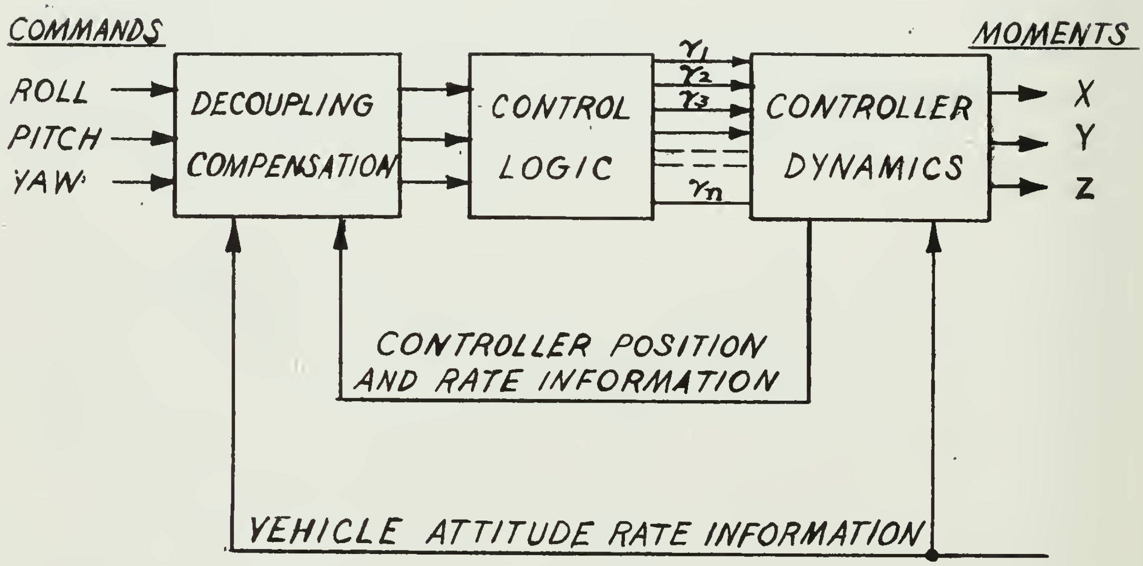


Figure 3.1 Block diagram of a spacecraft attitude control system showing provision for decoupling compensation and control logic for a controller with n degrees of freedom ($n \geqq 3$).

3.2 Moment Contribution of a Typical Control System

The moment contribution for a particular control system must be determined so that it can be substituted in the spacecraft equation of motion, equation 2.4.3. Assuming that Appendix F contains the moment contributions for the individual controllers making up the system, then it is merely necessary to sum the individual contributions to find that of the whole control system. The resulting matrix equation will be a function of p. q. and r which are the spacecraft attitude rate variables, and the variables representing the total degrees of freedom of all the controllers which are called the control system input variables. It is convenient to arrange the equations in an orderly matrix form so that the resulting expression is simplified as much as possible. Thus, suppose that a control system composed of several controllers whose combined degrees of freedom is n is defined. Then the moment contribution of this control system can be arranged in the following form.

$$\sum M_{\begin{array}{l}\text{Typical}\\ \text{Control}\\ \text{System}\end{array}}\Bigg|_A = \begin{bmatrix} \text{Control System} \\ \text{Input Matrix} \\ \\ (3 \times n) \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \vdots \\ \dot{\gamma}_n \end{bmatrix} + \begin{bmatrix} \text{Control System} \\ \text{Coupling Matrix} \\ \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq. 3.2.1)

The moment contribution is composed of the product of a 3 x n matrix called the control system input matrix times an n x 1 column matrix of the control system input variables, plus a square 3 x 3 matrix called the control system coupling matrix times a 3 x 1 column matrix of the vehicle attitude rate variables. Actually the control system input variables will appear as angular accelerations, but for pure gyro type controllers it is sufficient to say that this column matrix is simply the time rate of the input variables of the individual controllers making up the control system.

In the majority of cases and particularly when active control systems are to be considered the ultimate aim is to write the equations of the roll, pitch, and yaw of the vehicle in three independent equations. With this in mind two changes are made in equation 3. 2. 1 to arrive at a more simplified matrix equation. The first of these changes is that a substitution is made for the n x 1 column matrix of control system input variables and the substitution is of the form:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \\ \gamma_n \end{bmatrix} = \begin{bmatrix} \text{Control} \\ \text{Logic} \\ \text{Matrix} \\ (n \times 3) \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \qquad \text{(Eq. 3. 2. 2)}$$

Derivation of the elements of the control logic matrix will be presented in the next section, but briefly the control logic matrix is chosen so that when premultiplied by the control system input matrix the result is a 3 x 3 nearly diagonal matrix called the primary control matrix. The second change made in equation 3. 2. 1 is that the control system coupling matrix is factored into the sum of a diagonal matrix and an antisymmetric matrix, then the diagonal matrix is lumped with the terms of the vehicle that are fixed.

The above two changes will transform equation3. 2. 1 into a more suitable form as follows:

$$\sum M_{\substack{\text{Typical} \\ \text{Control} \\ \text{System}}} \Bigg|_A = \begin{bmatrix} \text{Primary} \\ \text{Control} \\ \text{Matrix} \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} \dot{n}_x \\ \dot{n}_y \\ \dot{n}_z \end{bmatrix} + \begin{bmatrix} \text{Gyroscopic} \\ \text{Coupling} \\ \text{Matrix} \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq. 3. 2. 3)}$$

In words equation 3. 2. 3 makes three new definitions as follows:

1. The three control system variables are defined as the primary control variables.

2. The 3 x 3 matrix operating on the primary control variables is called the primary control matrix and it is nearly diagonal.

3. The 3 x 3 matrix operating on the vehicle attitude rate variables is an antisymmetric matrix and is defined as the gyroscopic coupling matrix.

3.3 Control Logic Matrix

The purpose of defining a control logic matrix is to define three variables called, the primary control variables which give nearly non-interacting control over the vehicle attitude variables in roll, pitch, and yaw. This implies that the primary control matrix is diagonal, or nearly diagonal. It is clear that if the control system input matrix were a 3 x 3 coefficient matrix then the control system logic matrix would be the inverse of the control system input matrix. Since the control system input matrix is not composed of constant elements, in general, we cannot expect to arrive at a diagonal primary control matrix for all values of control system input variables. We can, however, choose a range of these variables over which the primary control matrix is nearly diagonal. Suppose then that we choose the null positions of the controllers as the center of a range of interest and evaluate the control system input matrix for zero angles. Under this constraint the control logic matrix is found to be a simplification of the transpose of the control system input matrix.

The justification for defining the control system logic matrix as being a simplification of the transpose of the control system input matrix evaluated for zero control system input variables is as follows: The physical geometry of the controllers when constrained to angular movements in a small range yield control moments which are resolved along three orthogonal axes; therefore the row vectors making up the control system input matrix for zero angles are orthogonal and an orthogonal matrix times its transpose gives a diagonal matrix.

In practice it is found that the simplest control logic matrix that can be derived is desirable. An example of the steps in arriving at control logic is given here to explain the procedure, and the reason we speak of a simplification of the transpose of the control system input matrix. Let us consider a control system such as the four controller system (12 - 34 - 1234) defined in

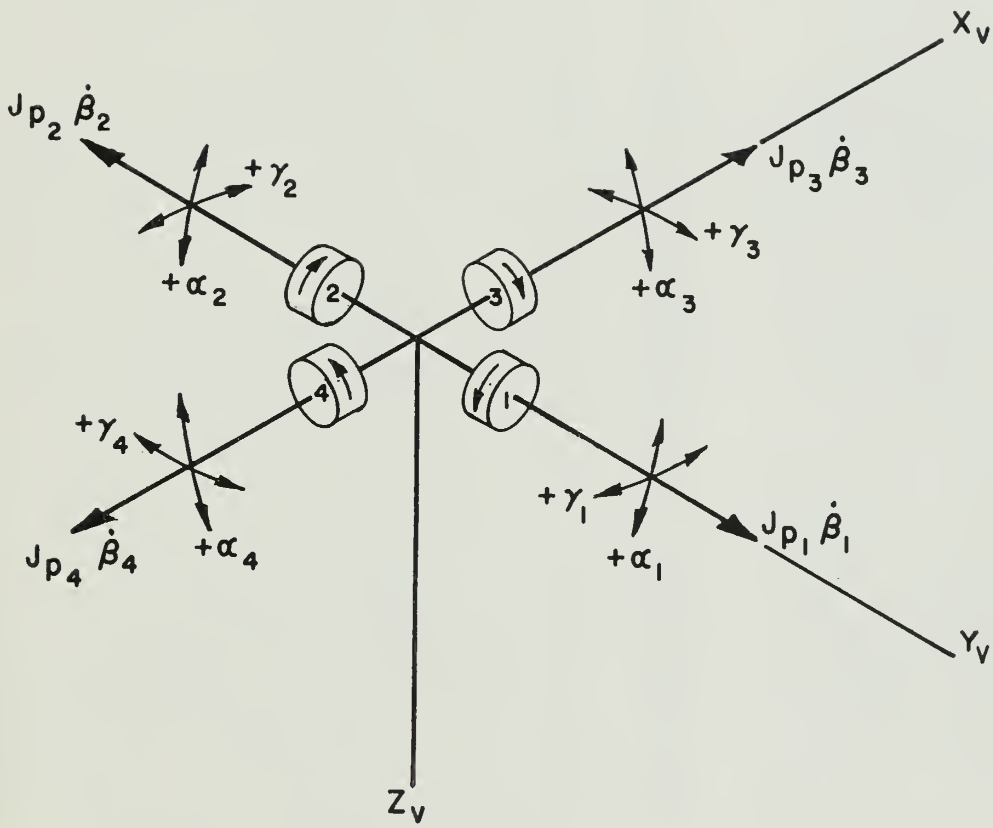


Figure 3.3. A Schematic Representation of a Four Controller System (12-34-1234). $+\dot{\alpha}_1$, $+\dot{\alpha}_2$, $+\dot{\alpha}_3$ and $+\dot{\alpha}_4$ gives yawing moments to the left. $+\dot{\gamma}_1$ and $-\dot{\gamma}_2$ gives rolling moments to the right. $+\dot{\gamma}_3$ and $-\dot{\gamma}_4$ gives pitching moments that tend to pitch nose down.

Appendix G and which uses controllers numbers 1 and 2 for roll control, 3 and 4 for pitch control, and all four controllers for yaw control. The controller configurations are shown in Figure 3. 3. Adding the moments contributed by each of the controllers from Appendix F (i. e. Equations F. 4. 1 through F. 4. 4) and arranging in a convenient matrix form as suggested by equation 3. 2. 1 gives for the control system input matrix of the system the following:

$$\begin{bmatrix} (-J_{p1}\dot{\beta}_1 C\alpha_1 C\gamma_1) & (J_{p2}\dot{\beta}_2 C\alpha_2 C\gamma_2) & (-J_{p3}\dot{\beta}_3 C\alpha_3 S\gamma_3) & (J_{p4}\dot{\beta}_4 S\gamma_4 C\alpha_4) \\ (-J_{p1}\dot{\beta}_1 C\alpha_1 S\gamma_1) & (J_{p2}\dot{\beta}_2 C\alpha_2 S\gamma_2) & (J_{p3}\dot{\beta}_3 C\alpha_3 C\gamma_3) & (-J_{p4}\dot{\beta}_4 C\gamma_4 C\alpha_4) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (J_{p1}\dot{\beta}_1 S\alpha_1 S\gamma_1) & (-J_{p2}\dot{\beta}_2 S\alpha_2 S\gamma_2) & (-J_{p3}\dot{\beta}_3 S\alpha_3 C\gamma_3) & (J_{p4}\dot{\beta}_4 S\alpha_4 C\gamma_4) \\ (-J_{p1}\dot{\beta}_1 S\alpha_1 C\gamma_1) & (J_{p2}\dot{\beta}_2 S\alpha_2 C\gamma_2) & (-J_{p3}\dot{\beta}_3 S\alpha_3 S\gamma_3) & (J_{p4}\dot{\beta}_4 C\alpha_4 S\gamma_4) \\ (J_{p1}\dot{\beta}_1 C\alpha_1) & (J_{p2}\dot{\beta}_2 C\alpha_2) & (J_{p3}\dot{\beta}_3 C\alpha_3) & (+J_{p4}\dot{\beta}_4 C\alpha_4) \end{bmatrix}$$

(Eq 3. 3. 1)

where the control system input variables have previously been arbitrarily chosen as an 8 x 1 column matrix

$$\begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dot{\gamma}_3 \\ \dot{\gamma}_4 \\ \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{bmatrix} \qquad \text{(Eq. 3.3.2)}$$

If we let each of the controllers have the same moment of inertia and require them to rotate at synchronous speeds then these constants can be removed from the matrix of equation 3.3.1 to give

$$J_p \dot{\beta} \begin{bmatrix} -C\alpha_1 C\gamma_1 & +C\alpha_2 C\gamma_2 & -C\alpha_3 S\gamma_3 & +C\alpha_4 S\gamma_4 & +S\alpha_1 S\gamma_1 & S\alpha_2 S\gamma_2 & -S\alpha_3 C\gamma_3 & S\alpha_4 C\gamma_4 \\ +C\alpha_1 S\gamma_1 & +C\alpha_2 S\gamma_2 & +C\alpha_3 C\gamma_3 & -C\alpha_4 C\gamma_4 & -S\alpha_1 C\gamma_1 & S\alpha_2 C\gamma_2 & -S\alpha_3 S\gamma_3 & S\alpha_4 S\gamma_4 \\ 0 & 0 & 0 & 0 & +C\alpha_1 & C\alpha_2 & C\alpha_3 & C\alpha_4 \end{bmatrix}$$

(Eq. 3.3.3)

Choosing the range around the null positions of the control system input variables gives for the above matrix (omitting the pre-multiplying constant)

$$\begin{bmatrix} -1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & +1 & +1 & +1 \end{bmatrix}$$

(Eq. 3. 3. 4)

Taking the transpose of this gives the control logic matrix which is written with the related variables as follows.

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & .1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

(Eq. 3. 3. 5)

To have some degree of uniformity we let positive rates of the primary control variables give roll to the right, pitch nose up, and yaw to the right respectively, but to arrive at this relationship since $+\dot{\gamma}_1$ yields roll to the right, $+\dot{\gamma}_4$ gives pitch nose up and $-\dot{\alpha}_1$ gives yaw to the right we change the sign of all the vectors of the control logic matrix of equation 3. 3. 5. This operation yields the desired form

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} +1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

(Eq. 3. 3. 6)

The control logic for various control systems are given in Appendix G.

Note that since each element for a particular column of the control logic matrix is multiplied by the same primary control variable that it is permissible to divide or multiply the elements of any column vector by a constant factor which is essentially what was done in going from equation 3. 3. 5 to equation 3. 3. 6.

In some systems the elements of the control system input matrix do not give simple integers as the above example, and it is usually desirable to simplify the control logic matrix by dividing (or multiplying) each column by a constant factor.

3.4 Primary Control Matrix

The primary control matrix (or simply control matrix) is defined as a 3 x 3 matrix of functions which operates on the primary control variables. If the units of the primary control variables are angular acceleration then the units of the elements of the primary control matrix are moments of inertia of the controlling elements of the control system, and the magnitude of these elements represent the effective value of the moment of inertia about the appropriate axes. This matrix could have been defined as an effective moment of inertia matrix, but moment of inertia is not a vector quantity. With systems whose controlling elements have equal polar moments of inertia it is convenient to factor out the common factor, the polar moment of inertia, and this leaves the units of the elements of the primary control matrix dimensionless. When this common factor is removed from the control matrix the elements of the matrix then represent a ratio of the effective moment of inertia about the various axes. Therefore, we loosely refer to the resulting matrix as the primary control matrix.

The primary control matrix is usually non-linear and contains angular functions of the primary control variables. However, in practice it is found that the control matrix has more physical interpretation if the elements are expressed in the angular variables of the controlling elements. The variables of the controlling elements are related to the three primary control variables by the control logic matrix. If we examine the primary control matrix for a small range of the primary control variables and evaluate the non-linear primary control matrix elements for the center values of the variables, then the primary control matrix can be represented as a coefficient matrix. That is, we linearize the equations with respect to the primary control variables for a particular small range. Thus with due consideration for the moments of inertia of the controller and

the spacecraft, the coefficients of the control matrix represent an index of control sensitivity for nine different situations corresponding to the nine elements of the 3 x 3 primary control matrix. These will be subsequently further broken down into diagonal effects and off-diagonal effects. We note also that for steady state conditions (vehicle attitude rates are zero) that the value of the coefficients of the control matrix represent the degree of saturation of the controlling elements. Let us consider the various elements of the primary control matrix to learn how these elements affect the spacecraft attitude control system.

First, examination of the primary control matrix shows that the terms on the diagonal of the matrix provide non-interacting control moments which we call primary control moments. Further, the off-diagonal terms of the control matrix contribute interacting control moments which we call cross-control moments. To have a control system which is completely non-interacting it is necessary for the control matrix to be a diagonal matrix with all terms that are off the diagonal to be zero so that all cross-control moments vanish. Note in this case that if the control matrix is considered to be made up of column vectors that each vector is composed of all zeros except the diagonal element. Then by inspection of a control matrix we can quickly determine which of the primary control variables contribute cross-control moments and which of them do not.

We do not rule out the case where cross-control moments are permitted and considered desirable. For example in the control of airplanes a limited amount of dihedral effect is desirable. However, certainly the diagonal terms must dominate and we leave judgment of the magnitude of the off-diagonal terms for comparison with the detail specifications for the control system which we are designing. In any case, if the cross-control moments are not negligible then they must be minimized either by normal closed loop performance of the control system,

or by providing a de-crossing control by some suitable logic. We define de-crossing control as a scheme for eliminating cross-control moments.

3.5 Gyroscopic Coupling Matrix

The gyroscopic coupling matrix is a 3 x 3 antisymmetric matrix which operates on the vehicle attitude rate variables. Actually, in the controller equations the matrix that operates on these variables is not antisymmetric but has diagonal elements.

It is convenient to separate this square matrix which operates on the spacecraft attitude variables into the sum of a diagonal matrix and an antisymmetric matrix. Then if we consider the diagonal matrix operating on the vehicle attitude rate variables we find that this moment contribution can be lumped with the spacecraft moment of inertia terms. Thus, the remaining square matrix operating on the vehicle attitude rate variables is an antisymmetric matrix, and it is this antisymmetric matrix that we choose to define as the gyroscopic coupling matrix. With the risk of repetition, we note that the gyroscopic coupling matrix is antisymmetric with zeros on the diagonal because we have removed the diagonal terms and lumped them with the spacecraft to increase by a small amount the moments of inertia of the spacecraft. The units of the elements of the gyroscopic coupling matrix represent angular momentum acting normal to the respective vehicle attitude variables; however, again for simplicity we choose to factor out the polar moment of inertia of the controller so that the units then remaining in the gyroscopic coupling matrix represent the angular velocity of the controllers normal to the respective vehicle attitude axes. To further simplify the gyroscopic coupling matrix, when there are constant factors in the matrix these factors are taken out front of the matrix, and it is found in all pure gyroscopic controllers when all controllers have the same angular velocity that the constant rotor speed can be factored out of the matrix.

The off-diagonal terms representing gyroscopic coupling of one axis of the spacecraft to another may under certain

conditions be useful torques; however, in general they are not useful and are considered undesirable. The moments arising from the gyroscopic coupling matrix must be tolerated or compensated for in order to decouple the gyroscopic moments in the same manner and for the same reasons that it was necessary to de-cross the cross-control terms arising from the off-diagonal terms of the control matrix. The terms of the gyroscopic coupling matrix are found to be functions of the primary control variables, and again if we linearize the gyroscopic coupling matrix for a small range of the primary control variables we find that the gyroscopic control matrix is an antisymmetric coefficient matrix the coefficients of which can be interpreted to yield the degree of saturation of the control system. This is explained in the following paragraphs.

Control systems which utilize the gyroscopic properties of a rotating mass can be classified into zero-momentum systems and non-zero-momentum systems. The zero-momentum system implies that with no initial saturation of the control system that when the spacecraft is non-rotating with respect to the 0 frame of reference, then the total angular momentum of all of the controlling elements has a zero resultant. The non-zero-momentum system naturally refers to the same conditions where the resultant is designed to be non-zero-angular momentum. Thus for zero-momentum control systems the coefficients of the linearized gyroscopic coupling matrix are zero and in the steady state any non-zero coefficient represents some saturation of the control system. For non-zero momentum systems the gyroscopic coupling matrix will have non-zero terms for the steady state non-saturated condition. To assist the discussion let us name the unsaturated steady state linearized matrices as initial matrices. With this definition then we must compare the gyroscopic coupling matrix for a non-zero-momentum control system with the initial gyroscopic coupling matrix in order to interpret the coefficients as indications of saturation.

In the non-steady state the coefficients of the gyroscopic coupling matrix represent cross coupling moments resulting from unit rates of roll, pitch, and yaw. In general, these gyroscopic coupled moments are not desirable (unless specifically otherwise designed for) and they must be either tolerated or decoupled by suitable compensation circuits.

3.6 Compensation to Isolate Control Loops

It has been shown that a spacecraft attitude control system may yield undesirable cross-control moments and undesirable gyroscopic coupling moments. A scheme for providing compensation to minimize these moments is called de-crossing and de-coupling respectively. Ideally, if de-crossing and de-coupling compensation is provided along with the compensation for the vehicle cross coupling terms the matrix equation for the spacecraft and its control system can be separated into three independent equations. Therefore, the purpose of compensation is to diagonalize the control matrix and null the gyroscopic coupling matrix. Some authors refer to this diagonalization as non-interaction of controls.

Consider first the control matrix and how we may derive the required compensation to diagonalize this matrix. If a control system possesses a control matrix which is not a pure diagonal matrix, but instead, has non-zero terms off the diagonal then the system exhibits cross-control response. This implies that a pure input about one axis will have not only a response about the input axis, but in addition, it will have a response about at least one other axis. One way we can correct a cross-moment is by making a change in the primary control variable having authority over the axis being affected by the cross moment. This by making a change in the primary control variable we can effectively compensate for a cross-moment. The result is the magnitude of the change we make is just sufficient to neutralize the cross-control moment. We are attempting to eliminate the off-diagonal terms of the control matrix. This can be effected by providing a compensation to the primary control variables. Thus, for an arbitrary control system with a control matrix

$$\begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix}$$

which operates on primary control variables $\begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$

then an expression for the moments that are applied to the spacecraft as a result of the addition of compensation of the cross-control moments is as follows

$$M_{\substack{\text{Cross} \\ \text{Control} \\ \text{Comp}}}\Bigg]_A = A \frac{F}{F^1} \begin{bmatrix} a_{xx} & o & o \\ o & a_{yy} & o \\ o & o & a_{zz} \end{bmatrix} \begin{bmatrix} o & -\frac{a_{xy}}{a_{xx}} & -\frac{a_{xz}}{a_{xx}} \\ -\frac{a_{yx}}{a_{yy}} & o & -\frac{a_{yz}}{a_{yy}} \\ -\frac{a_{zx}}{a_{zz}} & -\frac{a_{zy}}{a_{zz}} & o \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix}$$

(Eq. 3.6.1)

where A = constant multiplier appearing in vehicle equation preceding control matrix.

F = Torque motor transfer function

F^1 = instrumented equivalent of F

Whenprecisely instrumented the addition of this compensation ideally cancels all cross-control moments, and thus the overall effect is to diagonalize the control matrix. In the above product of matrices the matrix

$$\begin{bmatrix} 0 & -\frac{a_{xy}}{a_{xx}} & -\frac{a_{xz}}{a_{xx}} \\ -\frac{a_{yx}}{a_{yy}} & 0 & -\frac{a_{yz}}{a_{yy}} \\ -\frac{a_{zx}}{a_{zz}} & -\frac{a_{zy}}{a_{zz}} & 0 \end{bmatrix}$$

is defined as the de-crossing compensation matrix.

(Eq. 3.6.2)

In a similar manner if we have a gyroscopic coupling matrix

$$\begin{bmatrix} 0 & n & -m \\ -n & 0 & l \\ m & -l & 0 \end{bmatrix}$$

operating on vehicle attitude rate variables,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

and if the primary control matrix is, again,

$$\begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix}$$

Then the moment we require from the compensation is expressed by

$$\left.\sum M_{\substack{\text{Gyroscopic}\\ \text{Coupling}\\ \text{Compensation}}}\right]_A = A\frac{F}{F^1}\begin{bmatrix} a_{xx} & o & o \\ o & a_{yy} & o \\ o & o & a_{zz} \end{bmatrix}\begin{bmatrix} o & \frac{-n}{a_{xx}} & \frac{m}{a_{xx}} \\ \frac{n}{a_{yy}} & o & \frac{-1}{a_{yy}} \\ \frac{-m}{a_{zz}} & \frac{1}{a_{zz}} & o \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

A, F and F^1 same as defined for de-crossing compensation

(Eq 3. 6. 3)

Again we define the matrix

$$\begin{bmatrix} o & \frac{-n}{a_{xx}} & \frac{m}{a_{xx}} \\ \frac{n}{a_{yy}} & o & \frac{-1}{a_{yy}} \\ \frac{-m}{a_{zz}} & \frac{1}{a_{zz}} & o \end{bmatrix}$$

(Eq 3. 6. 4)

as a decoupling compensation matrix.

Ideally the compensation makes the off-diagonal terms zero, and since the gyroscopic coupling matrix is an antisymmetric matrix the final result is to ideally null the gyroscopic coupling matrix. By inspection of the de-crossing and de-coupling compensation matrices the designer has knowledge of the required compensation.

The compensation for the vehicle inertia cross-coupling moments is derived in a like manner. Thus, from equation 2. 4. 5 the vehicle inertia coupling matrix is

$$\begin{bmatrix} o & -r I_y & q I_z \\ r I_x & o & -p I_z \\ -q I_x & p I_y & o \end{bmatrix}$$

operating on vehicle attitude rate variables,

$$\begin{bmatrix} \dot{p} \\ q \\ r \end{bmatrix}$$

and if the primary control matrix is, again,

$$\begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix}$$

then the moment we require from the compensation is expressed by

$$\sum M_{\substack{\text{Vehicle}\\ \text{Inertia}\\ \text{Coupling}\\ \text{Compensation}}}\Bigg]_{A} = A^{F}_{F1} \begin{bmatrix} a_{xx} & o & o \\ o & a_{yy} & o \\ o & o & a_{zz} \end{bmatrix} \begin{bmatrix} o & +\frac{rI_y}{a_{xx}} & -\frac{qI_z}{a_{xx}} \\ -\frac{rI_x}{a_{yy}} & o & +\frac{pI_z}{a_{yy}} \\ +\frac{qI_x}{a_{zz}} & -\frac{pI_y}{a_{zz}} & o \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad \text{(Eq 3.6.5)}$$

Again we define the matrix

$$\begin{bmatrix} o & +\frac{rI_y}{a_{xx}} & -\frac{qI_z}{a_{xx}} \\ -\frac{rI_x}{a_{yy}} & o & +\frac{pI_z}{a_{yy}} \\ \frac{qI_x}{a_{zz}} & -\frac{pI_y}{a_{zz}} & o \end{bmatrix} \qquad \text{(Eq 3.6.6)}$$

as a vehicle inertia de-coupling matrix.

3.7 Summary

The procedures and techniques introduced in this chapter can be presented in a step by step form as follows.

1. Assuming that a particular control system is defined, form a matrix representation of the moment contribution for the complete control system by summing the contributions of the individual controllers. The contributions for the individual controllers can be found in Appendix F.

2. Arrange the equation in the following form.

$$\left.\sum M_{CS}\right\rfloor_A = \begin{bmatrix} \text{Control System} \\ \text{Input Matrix} \\ (3 \times n) \end{bmatrix} \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \vdots \\ \dot{\gamma}_n \end{bmatrix} + \begin{bmatrix} \text{Control System} \\ \text{Coupling Matrix} \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq 3.7.1)

3. Choose a control logic by the methods of section 3.3.

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix} = \begin{bmatrix} \text{Control} \\ \text{Logic} \\ \text{Matrix} \\ (n \times 3) \end{bmatrix} \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix} \qquad \text{(Eq 3.7.2)}$$

4. Substitute the control logic equation into equation 3.7.1 to arrive at the following form.

$$\left.\sum M_{CS}\right\rfloor_A = \begin{bmatrix} \text{Primary} \\ \text{Control} \\ \text{Matrix} \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix} + \begin{bmatrix} \text{Control System} \\ \text{Coupling Matrix} \\ (3 \times 3) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq 3.7.3)

5. If the control system coupling matrix contains diagonal terms neglect them by the justification that the moments of inertia of the control system is lumped with the vehicle. This gives the final form as follows.

$$\sum M_{CS}\Big]_A = \begin{bmatrix} \text{Primary} \\ \text{Control} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{\eta}_z \end{bmatrix} + \begin{bmatrix} \text{Gyroscopic} \\ \text{Coupling} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

(Eq 3.7.4)

6. Appendix G contains the equations for eighteen particular control systems. For each of these systems one can identify the elements of the primary control matrix and the gyroscopic coupling matrix, and thus the compensation defined by equations 3.6.2 and 3.6.4 can be found.

CHAPTER 4

TORQUE DISTURBANCES ACTING ON A SPACECRAFT

4.1 Introduction

In designing a control system for a vehicle one must have apriori knowledge of the torque disturbances likely to act on the spacecraft. The attitude control system must be adequate to compensate for maximum torques encountered plus have sufficient capability to give an adequate margin of control during the time the torques are in effect. The space environment in the close proximity to the earth has been probed by a large number of scientific rockets and satellites. The Explorer 1 [34] discovered the Van Allen radiation belt, and Explorers 3, 4, 6, 7, 10, and 12[35] contributed further data to define the Van Allen belts as well as to discover the draping effect of the geomagnetic field caused by the solar wind. The Pioneer series of scientific lunar probes extended the collection of experimental data to further distances from the earth and confirmed the results of the Explorer satellites. As reported by Wyckoff, [36] a recent Venus probe called the Mariner R had a mission to collect data on magnetic fields, high energy particles, cosmic dust, solar plasma, and radiometer soundings of the planet Venus. These experiments tell us a great deal about the space environment near the earth, and they give a little data on interplanetary space; however, most of the current technology of interplanetary space is based on experiments, observations, and theoretical derivations conducted on earth.

The torque disturbances that act on a spacecraft may be classified as either from external sources or internal sources. The external sources of torque arise from the space environment which is defined by the ambient force fields sourrounding the

spacecraft and the energy and mass particles which collide with the vehicle. A model of the space environment may be assumed by combining the effects of the Sun, the planets, and other interplanetary matter for each point in space. In discussing the space environment we first look at interplanetary space at a distance of about 1 to 1.5 astronomical distances from the sun neglecting the influences of the planets, following which, we then consider the effects of the earth.

Internal torque disturbances arise from mechanisms within the spacecraft, and these torques can be further divided into two sources analogous to the momentum exchange devices and the momentum transfer devices proposed for attitude control in section 1.1. Momentum exchange type disturbance torques arise from any change in angular momentum, measured with respect to the center of gravity of the spacecraft, of a mass that remains with the vehicle, and assuming that the mass is not being used for primary attitude control torques. A basic principle in the attitude control of a spacecraft is that all angular momentum within the vehicle must be either eliminated, minimized, or controlled for attitude control purposes. Thus all moving parts within the spacecraft, with the exception of those providing primary control torques, are classified as disturbance sources. In particular, the astronauts are a prime source of random disturbance torques.

The source of internal disturbances classified as momentum transfer in origin include all masses which are ejected from the spacecraft. Two torques of this nature are immediately apparent: mass expelled from the system inadvertently, such as gas leakage, and mass expelled from the system to provide a velocity change. A second basic principal in the attitude control of a spacecraft is that the torque disturbance resulting from a mass that is expelled from the spacecraft must have a zero mean with respect to the center of mass of the spacecraft, or if a zero mean is not possible, the mean torque should be minimized. Mass that is ejected

from the spacecraft for the purpose of desaturation of a momentum exchange type control system is not considered to be a disturbance torque. On the contrary, desaturation torque is necessary because of a previous non-zero mean torque disturbance, and for efficient mass expenditure the desaturation torque impulse per unit mass expended must be maximized. This can be accomplished by placing the desaturation control jets at the maximum possible distances from the center of mass of the spacecraft. The torque disturbances resulting from venting of fluid reservoirs aboard the spacecraft can be minimized by the use of equalizers located as close to the center of gravity as possible and with the axis of the equalizer perpendicular to the axis of maximum moment of inertia of the spacecraft.

4.2 Interplanetary Space Environment

As with the computation of a trajectory in orbital mechanics the perturbations of distant planets can usually be neglected in postulating a space environment for the purposes of defining torque disturbances on a spacecraft. Therefore, in this section we consider primarily the characteristics of space as a result of the sun plus a few of the smaller mass particles from the trails of comets. The region of interest is for the Mars excursion.

4. 2. 1 Interplanetary Matter

The nine major planets of the solar system are the largest bodies in orbit around the sun. In addition to these planets there are a great number of smaller bodies, particles, dust, and gases which constitute interplanetary matter. Interplanetary matter may be classified into six headings, the last of which is high energy particles. These particles contribute more to radiation hazards than to torque disturbances so they are discussed in section 4. 2. 3.

Asteroids
Comets
Meteroids
Extraterrestrial dust
Gas
High energy particles

The asteroids are minor planets of which a large number lie between the orbits of Mars and Jupiter, and they range in size from a mile to approximately 450 miles. More than 98 percent of all asteroids have perihelion distances beyond the orbit of Mars according to Jacchia[3] leaving ten asteroids to constitute a possible hazard on a flight to Mars. Since a collision with an asteroid would be catastrophic, their orbits must be plotted in relation to any planned spacecraft trajectory, so that there is a small likelihood of collision with a known asteroid. Because of their steeply inclined, highly eccentric orbits there are only a few known asteroids which may cause a conflict, so their effect on the launch window is expected to be negligible.

The comets are a mysterious breed of solar matter because of their appearance and construction. The comets are believed to consist of a nucleus, a nebulous coma, and a tail. The nucleus is small and round possibly of a few miles in diameter. The coma on the other hand is possibly 10^5 miles in size and is believed to be newly formed gas from the nucleus which is dispersed into the tail and wake of the comet. The tail of the comet measures millions of miles in length, and because of the effect

of the solar wind on the small particles making up the tail, the tail almost always points away from the sun. The brightness of a comet greatly increases as it gets closer to the sun because of the intense solar radiation effects on the cometary material causing vaporization and ionization of the particles. As with the asteroids, the comet wake is to be avoided in the planning of a mission until more knowledge is gained of the potential danger. Fortunately, like the asteroids, the comets follow highly inclined and eccentric orbits, and an encounter is not likely; nevertheless, the trajectory of any planned mission should be checked carefully to avoid conflicts with any known comets.

Much of the meteoric material and dust has its origin in the asteroids and comets. Meteoroids are bodies of low macroscopic density which travel in generally large and highly eccentric orbits about the sun. When a meteoroid strikes the Earth's atmosphere and leaves a visible trail, the effect produced is known as a meteor, and meteors are small enough to be completely destroyed before reaching the surface of the Earth. A body which successfully penetrates the atmosphere and strikes the Earth is called a meteorite. Meteorites which are too small to produce a meteor are called micrometeorites, and according to Vedder[37] their diameters are less than 1 mm. Interplanetary dust is considered to be composed mainly of tiny meteoric particles which may be as small as micron in diameter. Their presence in the plane of the ecliptic is evidenced by a phenomenon called zodiacal light which is a reflection of sunlight. By computing the force of gravational attraction for a spherical particle of matter and comparing this to the force caused by the radiation pressure acting on the cross section of the particle it is found that for particles of about 1 micron in diameter the force due to the solar pressure is greater than the gravitational attraction, and hence the particles will tend to be blown out of the solar system. This could be one source of interstellar dust which does not have a

great influence on the torque disturbances of a spacecraft, but the particles are worthy of mention because apparently they hold importance in the evolution of a galaxy such as the one containing the solar system. Oort[38] states that visual observations in the plane of the galaxy are limited to about one percent of the total surface of the galactic disc because of the fact that in the cool parts of the interstellar clouds some condensation has taken place. The small solid particles formed by this condensation obstruct observations by optical means to about five thousand light years, whereas the whole disk has a radius of about forty thousand light years. At lower frequencies using radio telescopes the range of observation is extended to perhaps five times the visible range. Astronomers such as Baker[39] call this absorption of visible light the Great Rift in the Milky Way and claim that the particles making up the interstellar dust are atoms and molecules of hydrogen, helium, and other gasses as well as very small solid particles of molecular size.

An additional source of interplanetary matter comes from sputtering of the surface of bodies, and the term is defined by Lucas on page 3-74 of reference 3. Lucas defines sputtering as the removal of atoms from a solid surface by bombardment of the surface with atoms or ions having kinetic energies of a few electron volts or more. In interplanetary space, sputtering will be caused by solar corpuscular radiation, and by collisions of a spacecraft with atoms of interplanetary gas. Close to earth, sputtering will be caused by collisions of a satellite with atoms of the earth's outer atmosphere and with particles of the Van Allen radiation belts. Sputtering can have an adverse effect on optical properties of materials and perhaps on the fatigue life of structural materials, although these effects have not been defined quantitively.

There have been a number of investigations of torque disturbances caused by meteoritic material; for example,

White[40] evaluated the meteoric effects on the attitude of an earth satellite. The quantity and energy distribution of micro-meteroids in space can be predicted by Whipple's Tables[41]. The results appear to be that the torque disturbances from the smaller particles which have a high probability of hitting the spacecraft are negligible, whereas the main difficulty with the larger particles is possible penetration of the spacecraft.

4.22 Interplanetary Magnetic Field

The presence of strong magnetic fields in the close proximity to the sun is evidenced by the Zeeman effect which is a splitting of the lines in the solar spectrum. Thus the sunspots are believed to produce powerful magnetic fields of the order of 3×10^{8} gamma (1 gamma = 10^{-5} gauss). The magnetic field between the earth and the sun was measured by Pioneer 5 and was found to vary between 3 and 50 gammas, the higher value being measured during a solar storm.

Hibbs on page 25-55 of reference 3 makes the following speculation about the interplanetary matter. "At the present time, it is believed that the interplanetary medium, in the neighborhood of the orbit of earth, is dominated by the solar wind. This wind consists of a stream of particles, primarily protons, moving radially outward from the sun, at speeds of 500 to 1000 km/sec, with a density of 10 to 100 particles/cm^3, in the vicinity of the earth's orbit." Since these particles are charged and therefore a conducting medium, it is presumed that they carry with them the magnetic field of the sun; thus, the magnetic lines of force from the sun are directed radially outward. During a solar flare it is believed that a large burst of high energy particles flow out from the sun, and it is not known how these particles interact with and modify the interplanetary magnetic field.

Kolcum[(35)] speaks about a solar wind of 1900 kilometers per second and finds reasons to believe that solar protons spiral around the magnetic field lines of the solar plasma cloud. A mechanism of this form would tend to support the belief that the radiation hazard in space is isotropic rather than directional in nature.

Magnetic fields will cause disturbing torques to be imparted to the spacecraft if the spacecraft possesses any magnetic moments which may arise from two sources: residual or per-

manent magnetic dipole moments of the spacecraft and electromagnetic dipole moments caused by circulating currents within the spacecraft. In the latter category we must include the induced currents caused by conductors cutting the magnetic lines of flux. Dipole moments caused by permanent magnets within the spacecraft are not expected to be large because the spacecraft will not contain any substantial amount of magnetic materials. Also, reasonable care in the design of electric wiring circuits in the spacecraft can minimize moments from electromagnetic sources. However, to arrive at a quantitative description of torques that can be derived from a magnetic field for purposes of desaturation consider a circular loop carrying a current. This loop will tend to align itself with the magnetic field so that the magnetic flux of the field passes normally through the loop. The torque produced is then given by

$$\overline{T} = \overline{M} \times \overline{B} \qquad \text{(Eq 4.22.1)}$$

where $\overline{M}$ is the magnetic moment of the loop

$\overline{M} = \bar{n} NIA$

$\bar{n}$ is the unit normal to the plane of the loop

N is the number of turns

I is the current in the loop

A is the Area of the loop

$\overline{B}$ is the magnetic induction of the field

The maximum moment produced by the loop per ampere turn is achieved when the magnetic induction vector lies in the plane of the loop and is given by

$$\frac{M}{NI} = AB \qquad \text{(Eq. 4.22.2)}$$

For a loop with a diameter of 10 meters in a field of 3 gamma

$$\frac{M}{NI} = 24 \times 10^{-8} \text{ kgm-m/ampere turn}$$

$$(1 \text{ kgm-m} = 9.807 \times 10^{7} \text{ dyne-cm.})$$

Such a small torque as derived above is completely negligible as far as disturbance torques are concerned, and unless the interplanetary magnetic field is considerably stronger than that measured by Pioneer 5, the field is not suitable for use in desaturating a momentum exchange type attitude control system.

4.2.3 Interplanetary Radiation

Radiation indigenous to space may originate from the sun or from sources outside of the solar system. Both sources radiate electromagnetic radiation over a wide spectrum and they both radiate high-energy particles. Cosmic radiation is a general term for high-speed particles which may consist of protons (80%), alpha particles which are helium nuclei (19%), and nuclei of heavier elements (1%). Infrared and larger-wavelength radiation present no problem to spacecraft design, and ultraviolet radiation is easily absorbed by glass panels. Cosmic radiation and x-ray radiation (wavelengths less than 100 angstrom) hold particular interest in space technology because of the biological hazards to man; however, since this radiation contains only 0.01 percent of the total energy there are negligible torque disturbances from this source. The solar spectrum between wavelengths of 2000A and 26,000A contains 97 percent of the total energy according to Katzoff[42], and the integrated intensity corresponding to a distance from the sun of one astronomical unit is 2.00 cal/cm^2/min or 1400 watts/m^2 which value is called the solar constant. For other radii the solar radiation is expressed by

$$S = S_{\oplus} \left(\frac{R_{\oplus}}{R}\right)^2 \qquad \text{(Eq 4.23.1)}$$

The pressure exerted by solar energy on a plate at an angle normal to the radiation is roughly given for a completely absorbing surface by

$$P = \frac{S}{C} \qquad \text{(Eq 4.23.2)}$$

where C = Speed of light = 3×10^8 m/sec

For one astronomical unit the solar pressure is found to be 4.67×10^{-5} dynes/cm^2 or 9.4×10^{-8} lbs/ft^2.

Evans[43] derives the normal pressure and tangential shear stress on an arbitrarily oriented intercepting plate which are as follows.

$$P = \frac{S}{C} \cos\theta \left[\left(\cos\theta + \frac{2}{3}\right) + \rho\,(1 - A)\left(\cos\theta - \frac{2}{3}\right) \right]$$

(Eq 4. 23. 3)

$$\tau = \frac{S}{C} \cos\theta \,\sin\theta \left[1 - \rho\,(1 - B) \right]$$

(Eq 4. 23. 4)

where S = Solar Constant given by equation 4. 23. 1

c = Speed of Light

θ = Angle between the normal to the plate and the radius vector to the sun

ρ = reflectivity

$$A = \frac{P_i - P_r/\rho}{P_i - P_w/\rho} \qquad B = \frac{\tau_i - \tau_r/\rho}{\tau_i}$$

For completely diffuse reflection

$$P_r = P_w,\ \tau_r = \tau_w,\ A = B = 1$$

$$P = \frac{S}{C} \cos\theta \left[\cos\theta + \frac{2}{3} \right]$$

(Eq 4. 23. 5)

$$\tau = \frac{S}{C} \cos\theta \,\sin\theta$$

For specular reflection

$$\frac{P_r}{\rho} = P_i,\ \frac{\tau_r}{\rho} = \tau_i,\ A = B = 0$$

$$P = \frac{S}{C} \cos\theta \left[(1 + \rho)\cos\theta + \frac{2}{3}(1 - \rho) \right]$$

(Eq 4. 23. 6)

$$\tau = \frac{S}{C} \cos\theta \,\sin\theta \left[1 - \rho \right]$$

(Eq 4. 23. 7)

To find the torque disturbances it is necessary to integrate over the entire external surface of the spacecraft in accordance with the following expressions.

In vector form:

$$\sum M_{\substack{\text{Solar}\\ \text{Radiation}}}\Big]_A = \int_{\text{Area}} \bar{l} \times \left\{ \tau \left[\frac{\bar{n} \times (\bar{n} \times \bar{l})}{\sin\theta} \right] - P\bar{n} \right\} da$$

(Eq. 4. 23. 8)

in matrix form

$$\sum M_{\substack{\text{Solar}\\ \text{Radiation}}}\Big]_A = \int_{\text{Area}} l\rfloor_A^{\star} \left\{ \tau \left[\frac{n\rfloor_A^{\star}\, (n\rfloor_A^{\star}\ l\rfloor_A)}{\sin\theta} \right] - P\, n\rfloor_A \right\} da$$

(Eq. 4. 23. 9)

where n is the unit outward vector normal to the surface of the spacecraft. This vector is a property of the spacecraft and expressed in matrix form n is written as n . See Appendix A. 3 for the special significance of the notation.

l is the vector from the center of mass to the element of surface area.

i is the unit vector aligned toward the sun and therefore is a function of the spacecraft attitude.

da is an element of surface area of the spacecraft.

To give an example of the magnitude of torque produced by solar radiation pressure consider that obtained from only one side of a circular umbrella of 100 meter radius and normal to rays of the sun.

$$\text{Area} = \frac{\pi R^2}{2} = 15,708\ \text{m}^2$$

$$\text{Pressure} = 0.467\ \text{dynes/m}^2$$

$$\text{Centroid of pressure} = \frac{4R}{3} = 0.424R = 42.4\ \text{m}$$

$$\text{Torque} = 0.317\ \text{kgm-meters}$$

For a 10 meter radius the torque would be 3.17×10^{-4} kgm-m or 2.2×10^{-3} lb-ft.

This simple example shows that solar radiation on large unsymmetrical areas can produce substantial torques; therefore, for the successful employment of an attitude control system of the momentum exchange type it is necessary to carefully balance the external torques attributed to incident radiation.

4.24 Interplanetary Gravitational Field

Newton's law of gravitation provides the fundamental basis for celestial mechanics which may be stated as: Every particle of matter in the universe attracts every other particle with a force that varies directly as the product of their masses, and inversely as the square of the distance between them. Although historically Kepler's laws preceeded Newton's universal law of gravitation, Kepler's laws can be derived from Newton's law. For a spacecraft that is remote from all planets the vehicle experiences a specific force of attraction toward the sun of the following magnitude:

$$f = \frac{E}{R^2} \qquad \text{(Eq. 4.26.1)}$$

where E is the solar gravitational parameter

$= 1.325 \times 10^{11} \text{ km}^3/\text{sec}^2$

R is the distance from the sun

$(1 \text{ AU} = 149.5 \times 10^6 \text{ km})$

A spacecraft in orbit about a planet, a moon, or the sun will be influenced by the mutual gravitational attraction of the spacecraft and the central body with perturbing effects from other bodies. The spacecraft like any other body responds to a total force which is the summation of the individual specific forces acting on each particle of the spacecraft where, by definition, specific force is the resultant of the gravitational attraction force and the inertia-reaction force for each unit mass.

The total force acting on the spacecraft together with the initial position and velocity vectors will largely determine the orbital parameters of the path followed by the spacecraft. However, when the vehicle has unequal principal moments of inertia, the spacecraft will experience a torque exerted about the center of mass of the vehicle. This torque arises because gravity fields

in space are not uniform, and since the torque is proportional to the gradient of the gravitational field it is called the "gravity gradient" torque. The sense of the torque is such as to tend to align the axis of minimum moment of inertia of the spacecraft with the direction of the gradient of the scalar gravitational field which is approximately towards the center of the central body about which the spacecraft is orbiting.

A simple explanation of the phenomenon of gravity gradient torque can be made by considering the earth-moon system. The earth and moon rotate about a common center called the barycenter, and because of the oceans the earth is not a rigid body. The gravity gradient effect on the earth is to bulge the ocean both towards the moon and also away from the moon thus giving the tidal period of approximately twelve hours. Along with this example one is reminded that the moon always faces the same side toward the earth which is strong evidence that the moon has unequal principal moments of inertia, and that the gravity gradient torque has provided the moon a rest point (see section 4.6).

Ogletree[21] gives for the gravity gradient the following moments

$$L_A = \frac{3E}{R^5} R_A \star J_A R_A \qquad \text{(Eq 4.26.2)}$$

If we consider a spacecraft aligned with its long axis toward the sun then the above expression may be reduced for small angles to the following:

$$L_A = \frac{3E}{R^3} \begin{bmatrix} (I_y - I_z) & A_Y A_Z \\ (I_z - I_x) & A_Y \\ (I_y - I_x) & A_Z \end{bmatrix} \qquad \text{(Eq. 4.26.3)}$$

This torque is seen to be inversely proportional to the cube of the distance from the sun and since the spacecraft is approximately one astronomical unit from the sun at the closest point, the torque is vanishingly small. This can be compared with the gravity gradient of the earth by looking at the magnitude of the factor $\frac{3E}{R^3}$. For the sun the value is $1.119 \times 10^{-13}/\text{sec}^2$ while for the earth at 1000 kilometers the value is $397/\text{sec}^2$. Although the x-component of equation 4.26.3 is seen to be a product of two small angles, Ogletree[21] shows that the first component when expressed in the Vehicle-Centered Planet Orbital Reference Frame, O, identically vanishes.

4.3 Influence of Earth

This section is restricted to a discussion of the influence of earth on the spacecraft because with the exception of gravity fields very little is known about the other planets. In orbital mechanics one can adopt a mathematical model of an earth within a sphere of influence and thereby neglect the effects of the sun since the sun's influence is reflected in the motion of the earth in space. This model is satisfactory for gravitational fields, but since the sun is the only active energy source within the solar system it holds a dominating influence over electromagnetic radiation. Accordingly the radiation incident on an earth satellite is predominately solar radiation which when combined with the radiation and reflection of solar radiation of the earth gives a complicated problem. While the earth reflected solar radiation may be significant for a satellite in a low orbit, the radiation of the earth is of a lower order and can be neglected in most problems.

The space environment at satellite altitudes differs from that of interplanetary space in its stronger gravity gradient, stronger magnetic fields, the presence of an atmosphere, and increased radiation in particular regions called the Van Allen belts. The solar radiation pressure is of the same order as in interplanetary space except for those occulated regions.

4.31 Geomagnetic Field

It is now generally believed that the geomagnetic field originates in a ferrous liquid core which circulates because of unequal radioactive heating. The resulting circulating currents create a giant electromagnet. A model of the earth's magnetic field may be visualized as that of a very strong magnetic dipole near the center of the earth. The south pole axis of the dipole lies in the northern hemisphere and intersects the surface of the earth at approximately 79^{o} North latitude and 69^{o} West longitude. Apparently the magnetic dipole axis does not pass

through the geometric center of the earth, but further refinements in description can be found elsewhere[44]. Near the surface of the earth there are numerous irregularities in the magnetic field, but these diminish at satellite altitudes. Beyond altitudes of about 5000 kilometers recent experimental satellites have shown that the geomagnetic field is influenced by solar radiation to give a draping effect described as a compression of the distance between the magnetic lines on the light side of the earth and an expansion of the lines of force on the dark side. The transition region between the solar wind and the terrestrial field is called the magnetopause and is visualized as a definite separation of the solar flux from the geomagnetic field. Further discussion of this draping effect can be found from Johnson[45], and the remainder of this section will consider the magnetic field at satellite altitudes.

The geomagnetic field is complicated by the fact that the dipole axis is skewed at an angle of approximately 11 degrees with respect to the earth's spin axis and rotates at earth rate. Because of this precession a satellite would have to be in an equatorial synchronous orbit in order to remain in a time-invarient magnetic field. In such an orbit the satellite with a controllable loop of current could obtain torques about only two axes since it is not possible to obtain any component of torque in a direction parallel to the magnetic field. For orbits other than the equatorial synchronous one, the magnetic field would be constantly changing and it would be necessary to either measure the field or to compute it, but it would be possible to generate control torques about the three attitude axes during at least a part of the orbit. Since these torques are available about only two axes for any one time it would be necessary to have some type of momentum exchange system aboard to achieve continuous three axis control. If the earth's magnetic field is approximated by a dipole, M, at the center of the earth, the magnetic field intensity vector in frame 0 has been determined by Sklar[21] to be as follows.

$$H_o = \frac{M}{\mu_0 R^3} \begin{bmatrix} -Q_6 \\ +Q_9 \\ -2Q_3 \end{bmatrix} \qquad \text{(Eq. 4.3.1)}$$

where $Q_3 = -C\mu_P S\lambda_K S\Lambda_M - S\mu_P C\Lambda_K C\lambda_K S\Lambda_M + S\mu_P S\Lambda_K C\Lambda_M$

$Q_6 = S\mu_P S\lambda_K S\Lambda_M - C\mu_P C\Lambda_K C\lambda_K S\Lambda_M + C\mu_P S\Lambda_K C\Lambda_M$

$Q_9 = S\Lambda_K C\lambda_K S\Lambda_M + C\Lambda_K C\Lambda_M$

$\mu_0 = 4\pi \times 10^{-7}$ Henries/Meter

M = Earths magnetic dipole moment

R = Radius of satellite from center of earth.

(See also Figure B. 13 of Appendix B.)

Even at satellite altitudes of 1000 nautical miles Ergin[46] shows the magnetic field with a strength of as much as 0.3 gauss (3×10^4 gamma). Using equation 4.22.2 for a circular loop 10 meters in diameter the torque available is 24×10^{-4} kgm-m/amp turn or 0.0174 lb-ft/amp turn which is a respectable torque. However, this is derived from an active loop of current. For spacecraft in which the magnetic dipole moments are minimized the disturbances caused by the earth's magnetic field are considered to be negligible.

One of the major discoveries of the past decade has been the Van Allen belts. These belts are large regions of high energy radiation found to completely encircle the earth in the plane of the equator. The high energy radiation is thought to originate from the sun and cosmic sources and has been trapped by the geomagnetic field. Because of the deleterious effects of the high energy particles on human life and some semi-conductor materials, the Van Allen belts are to be avoided although this type

of radiation does not present torque disturbances to complicate the control problem. It would appear that the Van Allen belts offer serious obstacles to the long period satellite following an equatorial orbit.

4.32 Aerodynamic Torques

Aerodynamic torques act on a spacecraft during launch, during re-entry, and at the perigee of a low satellite orbit. The large torques that may arise during launch and re-entry require specialized control systems distinctly different from momentum exchange systems and considerations of launch and re-entry control systems are not included in the objectives of this thesis. At satellite altitudes of 200 kilometers or more the atmosphere is highly rarefied so that the mean-free path between molecules is large. The large mean distance that a molecule travels between collisions permits the assumption that there is no interaction between the incident stream of molecules and the reflected molecules. The aerodynamic torques can then be calculated by considering separately the incident and reflected molecules. Evans[43] finds the pressure from incident and re-emitted flux of momenta as a complicated function of surface temperature and reflection coefficient.

$$P = \frac{PU^2}{2S^2}\left\{ e^{-(S \sin \eta)^2} \left[\frac{2 - \sigma'}{\sqrt{\pi}} (S \sin \eta) + \frac{\sigma'}{2}\sqrt{\frac{T_w}{T}} \right] + \left[1 + \operatorname{erf}(S \sin \eta) \right] \left\{ (2 - \sigma')\left[\frac{1}{2} + (S \sin \eta)^2 \right] + \frac{\sigma'}{2}\sqrt{\frac{\pi T_w}{T}} (S \sin \eta) \right\} \right\} \qquad \text{(Eq 4.32.1)}$$

The shear stress is directly proportional to the reflection coefficient σ, and is completely independent of the wall temperature T_w.

$$\tau = \frac{\sigma PU^2}{2\sqrt{\pi} S} (\cos \eta) \left\{ e^{-(S \sin \eta)^2} + \sqrt{\pi} (S \sin \eta) \left[1 + \operatorname{erf}(S \sin \eta) \right] \right\} \qquad \text{(Eq 4.32.2)}$$

where U = relative velocity vector

ρ = density of particles in space

S = Speed ratio = $\frac{\bar{\alpha}}{2} M$ where M = Mach No.

η = angle of incidence

erf(x) = probability integral

σ = tangential surface reflection coefficient $= \frac{\tau_i - \tau_r}{\tau_i}$

σ' = normal surface reflection coefficient $= \frac{p_i - p_r}{p_i - p_w}$

α = accomodation coefficient $= \frac{dE_i - dE_r}{dE_i - dE_w}$

T = temperature

To find the torque disturbances from aerodynamic normal pressure and shear stress equations 4.23.8, and 4.23.9 can be used as was done in section 4.23. Admittedly, the above method is a laborous method and is justified only for investigation requiring extreme accuracy. An approximate method to estimate pitch and yaw disturbances is based on the drag of a flat plate given by the following equations.

$$M_y = \int_{Area} q C_D \, z \, da \qquad \text{(Eq 4. 32. 3)}$$

$$M_z = \int_{Area} q C_D \, y \, da \qquad \text{(Eq 4. 32. 4)}$$

C_D = drag coefficient $= 2\,(2 - \sigma') \approx 2$

$q = \frac{1}{2} P U^2$

a = projected frontal area

Obviously with high speeds and appreciable densities the aerodynamic torques can become extremely large and cannot possibly be balanced by momentum exchange control systems. During unpowered flight of a spacecraft in dense air it is evident that some type of maneuvering control is required to enhance its landing flexibility. Attitude control to produce reasonable lift/drag ratios appears to be available by means of controllable plates or flaps, and it is possible that the industry will choose this approach. For purposes of this study, it is concluded that a symmetrical frontal profile is required to minimize the torque equations of this section although it is more precise to say that equations 4.32.3 and 4.32.4 must be vanishingly small.

4.33 Gravitation Field

Since all bodies in space are subject to Newton's Law of Gravitation, the field around the earth follows that about the sun except that the constant of proportionality includes the mass of earth instead of the sun. Thus, neglecting the effects of the oblateness of the earth, the specific force of attraction towards the earth is given by

$$f = \frac{E}{R^2} \qquad \text{(Eq 4.33.1)}$$

where $E = 3.98 \times 10^5 \ km^3/sec^2$

R = distance of satellite from center of earth. (Radius of earth is 6378 km)

For Mars

$E = 4.29 \times 10^4 \ km^3/sec^2$
(Radius of Mars is 3310 km)

for Venus

$E = 6.242 \times 10^5 \ km^3/sec^2$
(Radius of Venus is 6200 km)

The torque attributed to gravity gradient is given by equation 4.26.3, and thus the torque is inversely proportional to the cube of the distance to the center of the planet which means that the torque decreases rapidly with altitude. At best, the gravity gradient torques are poor stabilizing torques except for perhaps very long life, inert reflecting antennas. For an interplanetary spacecraft the gravity gradient torques are considered to be negligible disturbances.

4. 34 Reflection and Radiation from Planets

Most of the radiation incident on an earth satellite consists of direct solar radiation of the sun, the reflected solar radiation from the sunlit portion of the earth, and the earth's emitted infra-red radiation. The direct solar radiation was discussed in section 4. 25 and was seen to be a significant source of torques available to the spacecraft. Evans[43] shows that the emitted and reflected radiation from the earth at an altitude of 650 kilometers is approximately one order of magnitude less than that of the direct solar flux. For an unmanned satellite the shielding of one surface by another from both the aerodynamic wind and radiant flux presents considerable torques which are grossly unsymmetrical. For the interplanetary vehicle these effects are completely negligible.

4.4 Disturbances from within the Spacecraft

It has been shown in the previous sections that interplanetary space poses negligible torque disturbances from external sources provided a reasonable symmetry is initially designed in the spacecraft. Disturbances from within the spacecraft originate from virtually every moving particle within the vehicle, and in some cases these torques are much more difficult to minimize. It has been stated earlier that one of the principles of attitude control by momentum exchange devices is that angular momentum within the spacecraft must be eliminated, minimized, or used in the control scheme. We classify internal torque disturbances as either a momentum exchange type disturbance where no mass leaves the system, or a momentum transfer type disturbance where mass is expelled from the spacecraft. Both types of disturbances can be deterministic, but large random disturbances appear to result primarily from crew movements.

Internal disturbances in which no mass leaves the system can be easily and more efficiently handled by momentum exchange type attitude control systems; however, disturbances in which mass leaves the system may have a non-zero mean torque, and this will cause any momentum exchange system to accumulate some degree of saturation. After the control elements have been saturated by the expulsion of mass from the spacecraft it is necessary to expell additional mass to return the momentum exchange control elements to a useful configuration. This indicates that consideration should be given in the design to give possible leakage sources such geometry that the torque impulse from the leakage is minimized such as is done with intentional leakage by the use of equalizers in handling boil-off of cryogenic propellants.

4.41 Deterministic Disturbances

Deterministic torque disturbances are considered to originate from moving parts within the spacecraft that operate continuously or follow some periodic schedule. This includes electric motors, actuators, valves, pumps, power turbines, closely contained moving fluids, and so forth. While it may be feasible to use some of these disturbances in the primary control scheme (particularly the angular momentum of large power turbines), if they are not so employed then it is desirable to minimize their influence. This can be accomplished in many cases by contra-rotating designs. For example, an inertial guidance measurement unit usually contains three integrating gyros and three pendulus integrating gyro accelerometers. In the design of such a unit it would appear desirable to provide orientation of the units such that the total angular momentum is zero, and this is particularly true if the unit is used on a duty cycle as proposed by Draper[(47)] and Hovorka[(48)].

The solution for the problem of deterministic disturbances within a spacecraft is to provide "angular momentum control" during the design of a spacecraft in very much the same way that weight control has been practiced through the years in the aerospace industry in the construction of all flying vehicles. With reasonable care during the design phase there is no reason to believe that the residual deterministic angular momentum cannot be reduced to a small uncertain random angular momentum with zero mean which will be easily within the capability of the momentum exchange attitude control system.

4.42 Random Disturbances

While random torque disturbances may come from various sources, by far the largest and most interesting source is from the crew of the spacecraft. Crew disturbances can be classified as small disturbances or large disturbances depending on whether the astronaut is restrained or is moving about in the spacecraft.

To determine the disturbances that a crew member may impart to a spacecraft it is necessary to have some knowledge of his daily routine. Therefore, the first step in the analysis is to assume a daily schedule of events and to classify likely body movements and restraints during the events. The following schedule seems reasonable for a typical astronaut because it is not likely that the routine for the various crew members will differ substantially.

Event	Alloted Time	Probable Movements
Rest	8 hours	Restrained
Duty Station	6 hours	Restrained
Maintenance	3 hours	Restrained
Scientific Activity	2 hours	Restrained
Food	2 hours	Restrained
Recreation	2 hours	Unrestrained
Other	1 hour	Unrestrained

In this schedule restrained movements imply that for vehicles without artificial gravity the period of rest will be performed in a sleeping bag, and the other restrained duty will be performed with the astronaut securely attached to the spacecraft since it is not reasonable to assume that he will be able to perform exacting movements with his hands without being trussed to the vehicle. Unrestrained movements are primarily those in which the astronaut displaces himself from one location in the spacecraft to another, and these movements are seen to be vastly different from the restrained type.

The anthropometric relationships for the astronaut are taken for the 50 percentile male similar to those of Mayo on page 27-16 of reference 3.

The moments of inertia for the various appendages are estimated, a summary of which follows.

Element		Pivot Point	Wt.	Radius of Gyration	Moment of Inertia
Head		Neck	7 lbs.	0.2 ft.	0.28 lbs-ft^2
Body		Hip	110	1	110
Arm		Shoulder	8 ea	1	8
	Forearm	Elbow	4 ea	0.4	0.64
	Upper Arm	Shoulder	4 ea	0.4	0.64
Leg		Hip	20 ea	1.2	28.8
	Sitting knee height	Knee	8 ea	0.5	2
	Buttock knee length	Hip	12 ea	0.5	3
			173 lbs.		

4.421 Analysis of Small Motions

Assume that an element of the astronaut with a moment of inertia I_d is moved through an angle of θ_d in time t_1. A reasonable angular velocity distribution appears to be a sine-squared curve represented by

$$W_d = \frac{2\theta_d}{t_1} \sin^2 \frac{\pi}{t_1} t \quad (o \leqq t \leqq t_1) \qquad \text{(Eq 4.421.1)}$$

Now, the moment applied to the spacecraft is proportional to the time rate of change in angular momentum, and if the moment of inertia of the element is assumed constant the moment disturbance is given by

$$M_d = \left(\frac{\theta_d I_d}{t_1}\right) \left(\frac{2\pi}{t_1}\right) \sin \left(\frac{2\pi}{t_1}\right) t \qquad \text{(Eq 4.421.2)}$$

The Fourier Transform of the moment disturbance is given by

$$H(\omega) = \frac{\theta_d I_d}{t_1} \left(\frac{2\pi}{t_1}\right)^2 \left[\frac{1 - e^{-j\omega t_1}}{\frac{4\pi^2}{t_1^2} - \omega^2} \right] \qquad \text{(Eq 4.421.3)}$$

If it is assumed that the small motion torque disturbance occurs an average of k times per unit time, the power density spectrum for the disturbance can be found by passing a train of Poisson distributed impulses having a zero mean through a shaping filter. Lee(49) justifies this method and gives the input/output relations which are as follows

Poisson distributed impulses with zero mean, k per unit time → Shaping Filter $H(\omega)$ → Power Density Spectrum

$$\Phi_{dd}(\omega) = \Phi_{ii}(\omega) \left| H(\omega) \right|^2 \qquad \text{(Eq 4.421.4)}$$

$$\text{Now} \quad \Phi_{ii}(\omega) = \frac{k}{2\pi} \qquad \text{(Eq 4.421.5}$$

Therefore the power density spectrum for a small motion of the astronaut is

$$\Phi_{dd}(\omega) = 16\, k\pi^3 \left(\frac{\theta_d I_d}{t_1}\right)^2 \frac{(1 - \cos \omega t_1)}{(4\pi^2 - \omega^2 t_1^2)^2} \qquad \text{(Eq 4.421.6)}$$

The parameters θ_d and I_d will not vary appreciably for a fifty percentile male performing a specific task; however, the parameters k and t_1 surely will vary with individuals.

4.422 Analysis of Large Motions

The large motions occur only when the astronaut moves from one location in the spacecraft to another, but the large

motions assume more statistical value because large disturbances are applied to the vehicle. These disturbances can be analyzed by deterministic methods but to get an expression useful in comparing several systems let us derive a power density spectrum for the large motions. For this motion we assume an angular velocity distribution relative to the mass center of the vehicle that is uniform at a value of ω_d over a time interval of t_2 but bounded on each end with a ramp of length t_1. If the distance of the astronaut to the center of mass of the vehicle is l and his mass is M then the torque applied to the vehicle is given by

$$T(j\omega) = Ml^2\left(\frac{\omega_d}{t_1}\right)\left(\frac{1 - e^{-j\omega t_1} - e^{-j\omega(t_2 - t_1)} + e^{-j\omega t_2}}{j\omega}\right)$$

(Eq 4. 422. 1)

Following an identical procedure of the previous section the power density spectrum of the large disturbance is found to be

$$\Phi_{tt}(\omega) = \frac{4k}{\pi}\left(\frac{Ml^2\,\omega_d}{t_1}\right)^2\left(\frac{1 - \cos\omega t_1}{\omega^2}\right) \qquad \text{(Eq 4. 422. 2)}$$

where it has been assumed that the time $t_1 \ll t_2$. The mean square value of the torque can be determined as follows.

$$\overline{T^2} = \frac{2\,k\,M^2\,l^4\,\omega_d^{\,2}}{t_1} \qquad \text{(Eq 4. 422. 3)}$$

4.43 Effects of Fluids Within Spacecraft

Interplanetary spacecraft will require non-cryogenic fluids such as water, lubrication oil, auxiliary mono-propellants, oxygen, and various fluids to be used in life support systems. To attain the long duration missions contemplated many of the fluids will be reprocessed in systems having closed loops so that the mass of the fluids will be limited. Since these fluids will be

in small quantities and will be closely contained their only effect will be from angular momentum generated by the fluids. The angular momentum of a loop of fluid can be expressed as follows.

$$\bar{H} = \oint \dot{m}\, d\bar{s} \times \bar{r} \qquad \text{(Eq 4. 43. 1}$$

where $\dot{m}$ = mass flow rate

$\bar{r}$ = radius vector from element of fluid of length $d\bar{s}$ to any fixed point.

For rigid loops $\dot{m}$ is not a function of the contour of the loop, and since $\oint d\bar{s} \times \bar{r} = 2A$ the angular momentum of the fluid loop reduces to

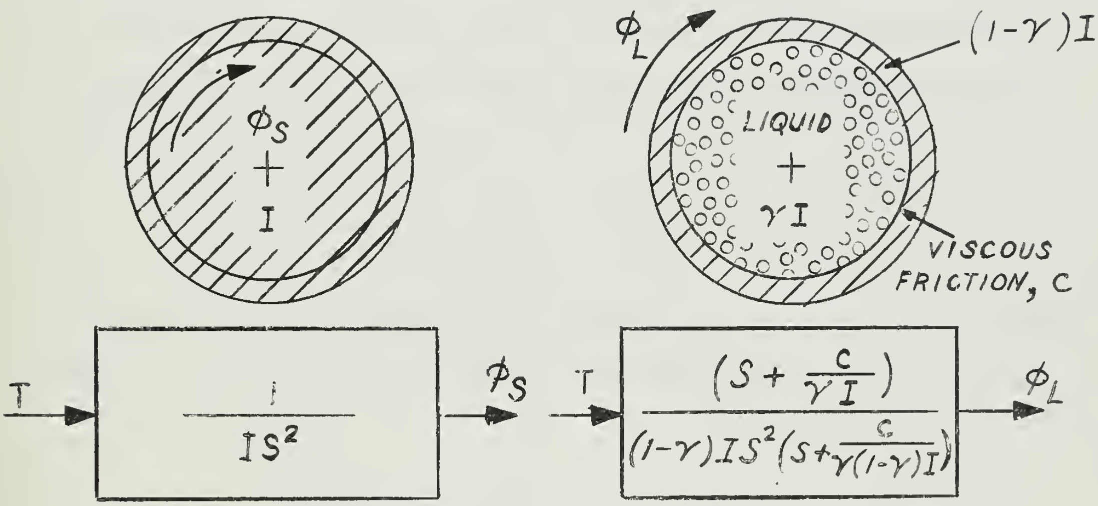
$$H = m\, 2A \qquad \text{(Eq 4. 43. 2)}$$

This expression implies that the angular momentum from a loop of fluid can be nulled if the projected integrated area of the loop is nulled.

Fluid loops can be considered for primary control purposes in which case it will be desired to maximize the area of the loop per total mass of fluid. This suggests that a circular loop of fluid is the most efficient. For a circular loop the angular momentum is given by the equation $H = M V r$ where M is the total mass, V the velocity of the fluid, and r the radius of the loop. Since viscous friction of the fluid will consume power, and viscous friction is some function of V, the equation $H = M V r$ is useful in determining trade-offs in the design of fluid systems for primary control purposes. Fluid controllers certainly have an appeal. For example, a loop of fluid of any density has a moment of inertia per mass ratio which approaches twice that attained with a rotating cylinder of any material. Fluid loops can be placed in low priority spaces within the spacecraft and can perform multipurpose functions such as thermal control of the spacecraft. Further consideration of fluid controllers is

beyond the scope of this work, but the use of fluids for primary control would appear attractive as a separate investigation. It has been suggested that the gyroscopic torques of rotating fluids can also be used as a primary control device.

If the spacecraft requires cyrogenic propellants then there will be large amounts of fluids on board. To illustrate how a large body of fluid affects the dynamic characteristics of a spacecraft consider the motion of a rocket with solid propellant compared to one with liquid propellant with both having the same maximum moment of inertia. Let us examine the roll displacement following a unit torque impulse, and assume the liquid rocket can be approximated by a model consisting of a cylinder rotating inside a hollow cylinder with simple viscous friction separating the two bodies.



The response to a unit torque impulse gives

$$\Phi_S = \left(\frac{1}{I}\right)t \qquad \text{(Eq 4.43.1)}$$

$$\Phi_L = \frac{\gamma^2}{c}\left(1 - e^{-\frac{ct}{\gamma(1-\gamma)I}}\right) + \left(\frac{1}{I}\right)t \qquad \text{(Eq 4.43.2)}$$

Therefore it is seen that the liquid rocket has a greater steady state roll displacement by the amount $\frac{\gamma^2}{c}$, but the angular rates of roll are the same, as they should be from the principle of angular momentum. During the transient period however, the rate of roll of the liquid propellant is greater than the solid propellant by the amount

$$\frac{\gamma t}{(1 - \gamma)I} e^{-\frac{ct}{\gamma(1 - \gamma)I}} .$$

Initially the propellant is not reacting to any of the applied impulsive torque and the effective moment of inertia has been reduced to the value $(1 - \gamma)I$.

A prime source of torque disturbances is gas leakage from the cabin of the spacecraft. In the MA-6 Mercury spacecraft the cabin pressure was 5.7 psi, and the results of this flight show cabin leakage to be of the order of 500 cc/min. It is difficult to estimate the torque disturbances without having more knowledge of the location of the leaks, but spacecraft designed for extended missions may require increased cabin pressures for crew comfort and the leakage problem may be increased. This again points to the initial spacecraft design to provide tightly sealed joints, and if possible, minimize the moment arm of any leakage that does occur.

4.5 Tracking

Besides torque disturbances which are applied directly to the spacecraft there are other inputs for which the spacecraft attitude control system must respond to give satisfactory performance. Strictly speaking these inputs are not disturbances but their effect is the same as if a disturbance were imposed on the control loop. Tracking disturbances occur whenever the spacecraft attitude control system attempts to track a line in space which does not have a uniform angular rotation (of course the tracking line can have a fixed spatial direction in which case there would be no tracking disturbance). Examples of this type disturbance are: velocity aberration resulting from tracking a star while in orbit about a central body, the tracking of a vertical line pointing to the mass center of a central body while in an eccentric orbit, the tracking of a landmark on the surface of a central body, and the tracking of a point on the earth while in a lunar orbit, or visa versa.

4.51 Velocity Aberration

Consider a spacecraft in a circular orbit about a central body with an orbital frequency of ω and let the spacecraft track a distant star. The aberration angle is found by dividing the component of velocity of the spacecraft that is normal to the line of sight to the star by the speed of light. Therefore, for a circular orbit inclined at angle i to the line of sight to the star the aberration angle is

$$\text{Aberration angle} = \frac{\omega r \sin \omega t \cos i}{C} \qquad \text{(Eq. 4.51.1)}$$

The aberration angle is thus a sinusoid at orbital frequency and has a maximum value of $\frac{\omega r}{C}$. For an earth orbit this would give a maximum angle of 2.62×10^{-5} radians and a maximum angular rate of the line of sight of about 3×10^{-8} radians/second.

4.52 Tracking of a Landmark on a Central Body

Let P be a point on the earth's surface and let S represent a point centered at the sensor aperture of a spacecraft in orbit about the earth. If O represents the center of the earth then the relative vector directed from S to P is

$$\bar{R}_{SP} = \bar{R}_{OP} - \bar{R}_{OS} \qquad \text{(Eq 4.52.1)}$$

This equation can be expressed in matrix form as the following.

$$R_{SP}\rfloor_I = Q_{IE}\, R_{OP}\rfloor_E - Q_{IP}\, R_{OS}\rfloor_P \qquad \text{(Eq 4.52.2)}$$

The relationship between angular rotation of a radius vector and the time rate of change of the vector relative to the axis of rotation is given by

$$\bar{W} \times \bar{R} = \dot{\bar{R}} \qquad \text{(Eq 4.52.3)}$$

Then given $\bar{R}$ and its time derivative one may solve for $\bar{W}$ under the constraint that $\bar{W} \cdot \bar{R} = O$. This gives for the above case the equation for angular rotation of the line of sight from the spacecraft to a landmark on the surface of the earth.

$$W_{SP}\rfloor_I = \frac{\left[R_{SP}\rfloor_I^{*} \quad \dot{R}_{SP}\rfloor_I \right]}{\left[R_{SP}\rfloor_I^{T} \quad R_{SP}\rfloor_I \right]} \qquad \text{(Eq 4.52.4)}$$

Consider the rotation that results from tracking a fixed point on the equator while in a low circular retregrade equatorial orbit. Thus for a 100 mile orbit the relative speed normal to the line of sight will be approximately 17,500 miles per hour which gives a peak value of angular rotation of the line of sight of 0.0515 rad/second. For a direct orbit the rotation is 0.046 rad/second.

4.53 Tracking the Vertical to a Central Body

In a similiar development to the preceeding section the angular rotation of the vertical to the central body is given by the following.

$$W_{OP} = \frac{\left.R_{OP}\right]_I^{*} \left.\dot{R}_{OP}\right]_I}{\left[\left.R_{OP}\right]_I\right]^T \left[\left.R_{OP}\right]_I\right]} \qquad \text{(Eq. 4.53.1)}$$

To get an approximation of the order of magnitude of this rotation consider the case of a velocity at perigee of 25,000 miles per hour at a distance from the center of the earth of 4100 miles. This gives an angular rotation of the vertical of approximately 0.0017 radians/second.

4.54 Tracking of a Point on the Earth while in a Lunar Orbit

The tracking of a point on the surface of the earth while executing a lunar orbit becomes more complex than the previous two sections and the reference frames necessary to define this motion have not been defined in Appendix B. However, we can place an upper limit on this motion by considering all motions in one plane. Orbital speed about the moon is approximately 3,400 miles per hour and if the earth point is moving 1,000 miles per hour in the opposite direction this gives a relative speed normal to the line of sight of 4,400 miles per hour separated by a radius of approximately 235,000 miles. The angular velocity of the line of sight is 5.2×10^{-6} rad/second.

4.6 Concept of a Rest Point for a Spacecraft

The successful employment of momentum exchange type attitude control systems requires that the external torque disturbances have a small mean value. Disturbances resulting from the movement of masses which remain within the spacecraft do have a zero mean. The mean torque disturbance from all other sources must be minimized by suitable spacecraft design and operation. The torque disturbances from masses which are ejected from the spacecraft usually are not dependent on the spacecraft attitude; however, it is probable that torques arising from ambient fields are some function of the attitude of the spacecraft. Accordingly, for each attitude of the spacecraft there exists a finite resultant torque which from a practical consideration is a continuous function of the attitude variables. Of course it is likely that a particular orientation of the spacecraft is required to accomplish the mission, but let us consider the case wherein the spacecraft attitude is not dictated and the spacecraft may be allowed to seek a position wherein the torque disturbances are an absolute minimum.

Let us first examine individually the torques resulting from a long thin spacecraft with a large solar sail at the extreme of the vehicle with the plane of the sail normal to the longitudinal axis of the spacecraft. Further let us assume that the spacecraft has a magnetic dipole moment with its axis coincident with the spacecraft. Now let us consider the torques acting on the spacecraft for a single degree of freedom in yaw, and let the reference direction for zero yaw be a line directed toward the sun. We neglect any effects of planets. Such a spacecraft would display torques from the individual sources as plotted in a qualitative manner as follows

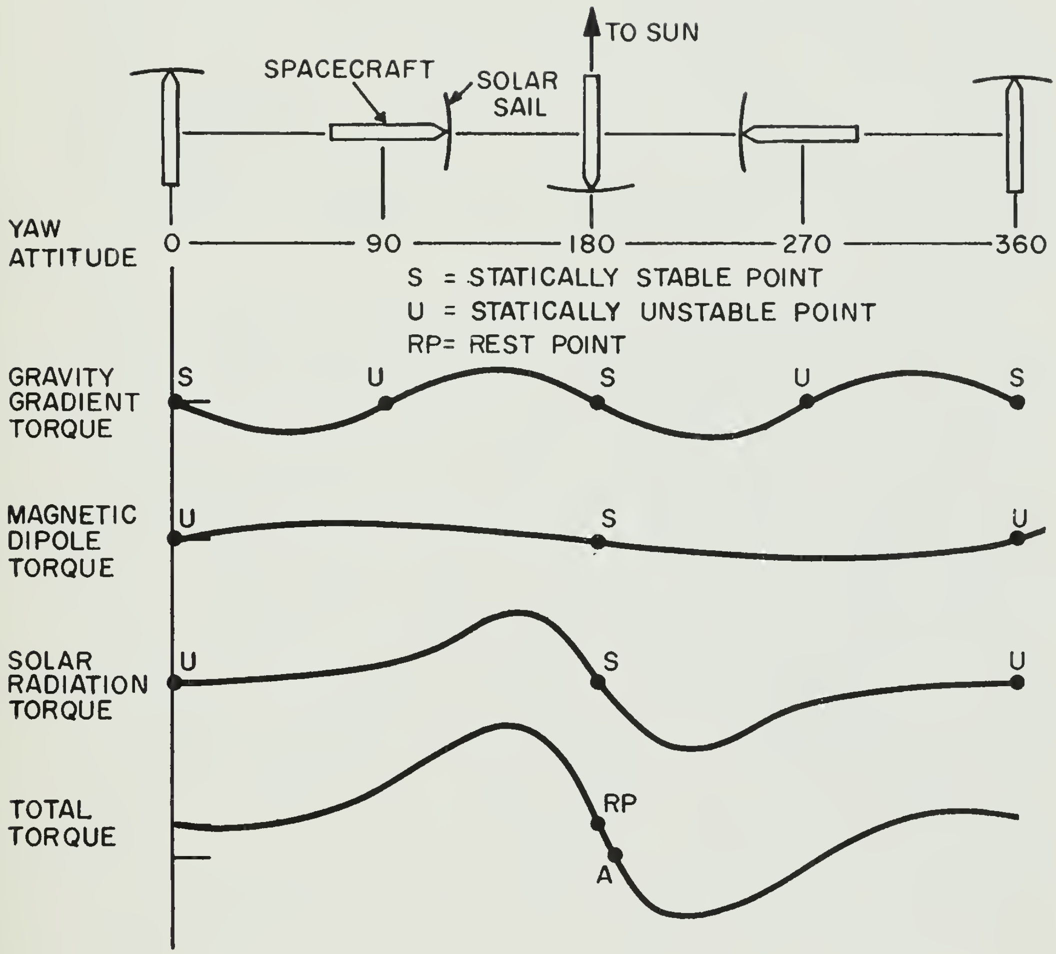


Figure 4.6.1 Illustration of a rest point for a spacecraft.

This example shows that a single rest point exists at a yaw attitude of 180 degrees. Damping characteristics in yaw cannot be shown in a plot of static stability characteristics such as the above simple example, but it has been stated earlier that damping is extremely small for all types of spacecraft and we must

depend on active methods of providing damping. Therefore, we seek only the rest position of the spacecraft so that momentum exchange type control systems can provide satisfactory attitude control with little expenditure of mass to compensate for non-zero mean external torques. Note how escaping fluids from the spacecraft would change the above rest position. Of course it is likely that escaping fluids are dependent on attitude, but let us assume that the torque from a leakage of cabin pressure is not attitude dependent. Therefore if this torque is a small positive value then it will shift the rest position to point A of Figure 4.6.1.

The above example is a simple one to illustrate the point. When we consider that the attitude of the spacecraft is a function of three variables, and that the torques from the various sources may not have any symmetrically coincident points, then the problem is considerably more complex. Nevertheless, there will still exist one or more rest points for which it may be possible to maintain the attitude to minimize expenditure of fuel for desaturation of a momentum exchange control system. Assuming that the total torque acting on a spacecraft can be expressed in three variables, the attitude variables in roll, pitch and yaw, then we may seek to determine the rest points from this function. At this point we may define the term <u>rest point</u> as the attitude of a spacecraft which results in zero applied torque and at which attitude the vehicle exhibits stable static stability characteristics. Because of the small magnitude and large uncertainty in the applied torques it may not be possible to compute the exact rest point for a spacecraft. However, in practice if a rest point exists, and one would exist provided the torques from external sources exceeded the torques caused by escaping fluids from the spacecraft, and if the rest point were a satisfactory attitude for extended spacecraft operation, then it would appear reasonable that the rest point could be located by a systematic trial and error adjustment in the attitude. The

error sensing element in this procedure will be the amount of saturation of the momentum exchange control elements. If only a single degree of freedom in yaw is considered this procedure is similar to that of bracketing a radio beam in the radio navigation of an aircraft. The aircraft intercepts a beam of known direction but because of drift due to winds and errors in the yaw indicating system in the aircraft as well as possible errors in beam alignment, the beam is not followed precisely when the aircraft is steered to the published beam heading. Accordingly, the aircraft will drift from the beam centerline, and the error is detected by a visual or aural device. After it is determined that the aircraft has drifted from the centerline of the beam, the pilot commands a correction in yaw of perhaps 20°. If 20° is not sufficient the aircraft is yawed another 20° or more until the beam is again intercepted. If, however, the 20° returns the aircraft to the centerline of the beam, upon reaching the centerline the pilot corrects 10° in a direction opposite to the initial 20° correction. This procedure is continued by taking one-half of the correction each time until finally the aircraft is following the beam very precisely. Although the spacecraft problem of finding the rest point is more complicated because of the three degrees of freedom, there is an error detection capability for all three control axes; therefore, it is considered that a similar bracketing procedure could be satisfactory in locating a rest point provided a nominal rest point were first known.

4.7 Summary of Disturbances

This chapter has attempted to summarize the torque disturbances that act on a spacecraft in regions of space remote from the effects of planets where the sun is dominant and in the vicinity of a planet where the sun still remains dominant in some respects. It was found that the torque disturbances in interplanetary space arising from sources external to the vehicle are extremely small if reasonable symmetry is designed into the spacecraft, and if the spacecraft is oriented toward the sun so that a symmetrical reflecting and absorbing surface is presented to the sun. Internal disturbances from the movement of the crew and other masses within the vehicle are present during all phases of the mission and can be significant disturbances.

In the vicinity of the earth the radiation from the sun persists in addition to aerodynamic torques, torques from magnetic interaction with the earth's field, gravity gradient torques, and sizable tracking disturbances as defined in section 4.5. A precise summation of all of these torques becomes prohibitively difficult, but fortunately the torques are small and many can be approximated by choosing their upper limit. A significant point in examining all of the above torques is that proper design and operation of the spacecraft can reduce the mean value of the torques to a very small magnitude so that the spacecraft can adequately be controlled by momentum exchange type control systems with infrequent desaturation requirements.

CHAPTER 5
CONSIDERATIONS IN THE CHOICE OF A SPACECRAFT ATTITUDE CONTROL SYSTEM

5.1 Introduction

In the previous sections there has been presented a substantial amount of information on the mission of the spacecraft, the characteristic equations of momentum exchange type attitude control systems, and the expected disturbances imposed on the vehicle. With such knowledge it should now be possible to choose a suitable spacecraft attitude control system to give maximum reliability while minimizing power and weight. The mission profile together with the payload to be carried dictates to a great extent the size and shape of a spacecraft. The systems chosen to be placed aboard the spacecraft will usually be optimum in some sense, so that they give the best and most reliable performance with the least weight and power consumption. This thesis is concerned primarily with extended missions which have durations of 400 days or longer in an interplanetary environment. The arguments of the previous chapter show that the external torque disturbances to which the vehicle is subjected are extremely small if reasonable symmetry is designed into the vehicle, and if the spacecraft is operated symmetrically to the disturbances. Also, it was determined that internal disturbances arising from the motion of masses within the spacecraft can be minimized, and in any event, the torque disturbances have a zero mean value. Accordingly, it is reasonable to choose some type of momentum exchange type control system to handle the spacecraft attitude control requirements.

The choice of a momentum exchange type attitude control system over a mass expulsion system can be justified in a number of ways despite the excellent performance achieved from gas jet systems. The fact that minimum ejection of fuel mass is a

prime requirement in the choice of a control system along with the fact that the possibility of exhaustion of control fuel substantially reduces the reliability of the control system should be sufficient. Nevertheless, one could still argue that a gas jet system can give better results despite the ejection of mass over long periods of time. Therefore, to present a valid argument for the momentum exchange system over the momentum transfer systems it is necessary to make some sort of weight comparison which will be done in this chapter.

There appear to be at least three capabilities of mass expulsion systems that cannot be met by the momentum exchange type systems. These are the capability of mass expulsion systems to control large spinning vehicles (with large angular momentum vector), the ability to generate large rates of attitude change, and the ability to control a vehicle subjected to non-zero mean torques. Therefore, when we choose a momentum exchange type attitude control system we necessarily restrict ourselves in these three capabilities.

5.2 Reliability

The reliability of a control system is determined primarily by the mean time to failure of the individual components of the system and the redundancy in the system. For systems constructed of identical elements the reliability is dependent entirely on the redundancy provided, and of course the control scheme. Consider a comparison of several gyro type control systems defined in Appendix G. In this comparison it is assumed that the probability of failure of a single gyro controller is the same for all systems and is equal to P where P is a small number compared to one.

Table 5.2 Reliability of Several Momentum Exchange Attitude Control Systems Using Gyro Controllers

System Description	Redundancy	Prob of Losing Precise Control (2)	Prob of Losing 3-Axis Control (3)	Prob of Losing Sun-Pointing (4)	Prob of Losing Sun-Pointing with min. Power (5)
Three Controllers Orthogonal (5-3-1) (7)	None	3P	3P	2P	2P
Three Controllers Zero Momentum (78-78-1) (6)	None	3P	3P	2P	2P
Three Controllers Zero Momentum (178-78-178) (6)	One	3P	$3P^2$	$3P^2$	2P
Four Controllers (12-34-1234)	Any One	4P	$6P^2$	$5P^2$	2P
Six Controllers (56-34-12)	Any one per axis but total of only one	6P	$5P^2$	$10P^2$	4P

Notes: (1) See Appendix G. 2

(2) Probability of Losing Precise Control is the probability of losing any one controller of the control system.

(3) Probability of Losing 3-Axis Control is the probability of being incapable of controlling all three axes in any manner.

(4) Probability of losing Sun-Pointing Mode is the probability that the control system is incapable of controlling pitch and yaw.

(5) Probability of Losing Sun-Pointing Mode with minimum power is the probability of the control system being incapable of controlling pitch and yaw with the controllers normally used for roll control inoperative.

(6) Requires special control logic, controllers capable of being repositioned through large angles, and expenditure of mass to convert zero momentum system to orthogonal system.

(7) Requires expenditure of mass to disable roll controller.

Table 5. 2 shows the merit of having redundant controllers in the system since the probability of losing 3-axis control is 3P for 3 controllers versus $6P^2$ for 4 controllers. For a P of 0. 01 the numbers would be 0. 03 versus 0. 0006 or two orders of magnitude difference in probability of losing 3-axis control. The 4-controller system and the 6-controller system represent an arrangement of controllers which give inherent adaptive control since both of the systems respond to give uninterrupted control in case of the loss of angular momentum in any one controller. On the basis of the probabilities in the above table it would appear that both the 4-controller system and the 6-controller system should be further considered, but any system containing only three controllers should be disqualified because of poor reliability.

A parallel comparison for inertia reaction wheel control

systems shows similiar results, that is, redundant wheels must be provided for maximum reliability. It is the usual practice for sophisticated control systems to provide a large wheel on each axis for coars control and a smaller wheel for fine control. Such an arrangement of two wheels per axis will provide the desired system reliability although it may be necessary to increase the angular momentum capacity of the smaller wheels.

5.3 Average Power Consumption

It is shown later in this section that the control power for gyro controllers is small and can be neglected when compared to the power necessary to maintain the gyro at a constant angular spin rotation. The average power consumption of a gyro controller is then approximately constant, and for a complete control system the average power is directly proportional to the number of identical controllers in the system. Therefore, if Q represents the power to drive a single controller at operating speed, the following table compares the power for several momentum exchange systems under consideration.

Table 5.3 clearly shows the average power required by the four controller system is less than that for the six controller system. The three controller systems require special logic control, controllers capable of repositioning through large angles, and expenditure of mass to be competitive with the four controller system.

Table 5.3 Average Power Consumption Comparison for Several Attitude Control Systems

System Description	3-Axis Control Using All Controllers	3-Axis Control One gyro has failed	Sun Pointing Mode	Sun Pointing with One gyro Failed
Three Controllers Orthogonal (5-3-1)	3Q	impossible using SDF gyros	2Q[3]	2Q[2]
Three Controllers Zero Momentum (78-78-1)	3Q	impossible using SDF gyros	2Q[4]	2Q[4]
Three Controllers Zero Momentum (178-78-178)	3Q	2Q[1]	2Q[4]	2Q[4]
Four Controllers (12-34-1234)	4Q	3Q	2Q	3Q
Six Controllers (56-34-12).	6Q	5Q	4Q	5Q

(See Notes on next page.)

Notes:

(1) Requires special control logic, controllers capable of being repositioned through large angles, and expenditure of mass to convert zero momentum system to orthogonal system.

(2) This power is applicable only if the failed controller happens to be the roll gyro, otherwise sun-pointing is not possible without special logic and with controllers capable of being repositioned through large angles.

(3) Requires mass ejection to disable roll controller.

(4) Requires special control logic and gyros capable of being repositioned through large angles.

5.31 Comparison of Control Energy and Power for Inertia Reaction Wheel Systems Versus Gyro Type Systems

A simple approach to a determination of the energy required by an inertia reaction wheel system is to evaluate the kinetic energy of a wheel and vehicle which have zero initial energy, and assume that an ideal motor with zero losses drives the wheel.

$$E_w = \frac{1}{2} J_w \Omega_w^2 + \frac{1}{2} I_x p^2 \qquad \text{(Eq. 5.31.1)}$$

where J_w, I_x are moments of inertia of wheel and vehicle respectively.

Ω_w, p are the respective angular rates.

Equation 5.31.1 may be written in terms of the vehicle roll rate by requiring $J_w \Omega_w = I_x p$. This gives

$$E_w = \frac{1}{2} \frac{I_x^2}{J_w} p^2 \left(1 + \frac{J_w}{I_x}\right) \qquad \text{(Eq. 5.31.2)}$$

A measure of the ideal efficiency of a reaction wheel can be stated as the ratio of energy imparted to the vehicle to the total energy expended giving the following.

$$\text{Wheel efficiency} = \frac{J_w}{I_x + J_w} \qquad \text{(Eq. 5.31.3)}$$

In practice the ratio J_w/I_x is of the order of 10^{-3} or less so that equation 5.31.2 can be approximated by simply

$$E_w = \frac{1}{2} \frac{I_x^2}{J_w} p^2 \qquad \text{(Eq. 5.31.4)}$$

For the gyro controller let us consider a single degree of freeom in roll which for a pair of gyros in steady state conditions the equation 6. 2. 16 applies.

$$p = \frac{M}{2H \cos \gamma_1} \qquad \text{(Eq. 5. 31. 5)}$$

where M is the moment applied to the gyro

H is the angular momentum on a single gyro

γ_1 is the gimbal angle.

If no external moments are applied to the vehicle to change its total momentum then

$$I_x p = 2H \sin \gamma_1 \qquad \text{(Eq. 5. 31. 6)}$$

Since the gyro controller must be continuously torqued during a maneuver the energy expended can best be found by a time integration of the electrical power supplied to the torque motor.

$$E_G = \int_0^t P_G \, dt \qquad \text{(Eq. 5. 31. 7)}$$

The electrical power supplied to a permanent magnet direct current torque motor with no mechanical power output is given by the copper losses in the armature, and the torque produced by this motor is directly proportional to the current. Therefore,

$$P_G = \frac{R_a}{k^2} M^2 \qquad \text{(Eq. 5. 31. 8)}$$

where R_a is the armature resistance, ohms.

k is the sensitivity, lbs ft torque/amp.

Substituting equations 5. 3. 5 and 5. 31. 6 into this expression for gyro torque motor power required gives

$$P_G = \left[\frac{R_a I_x^2}{k^2}\, p_{max}^4\right] \frac{p^2}{p_{max}^2} \left[1 - \frac{p^2}{p_{max}^2}\right]$$

(Eq. 5.31.9)

where $p_{max} = \frac{2H}{I_x}$

For a given system all of the factors in equation 5.31.9 are constant except the roll rate, p, therefore the control power may be plotted as shown in Figure 5.31.1.

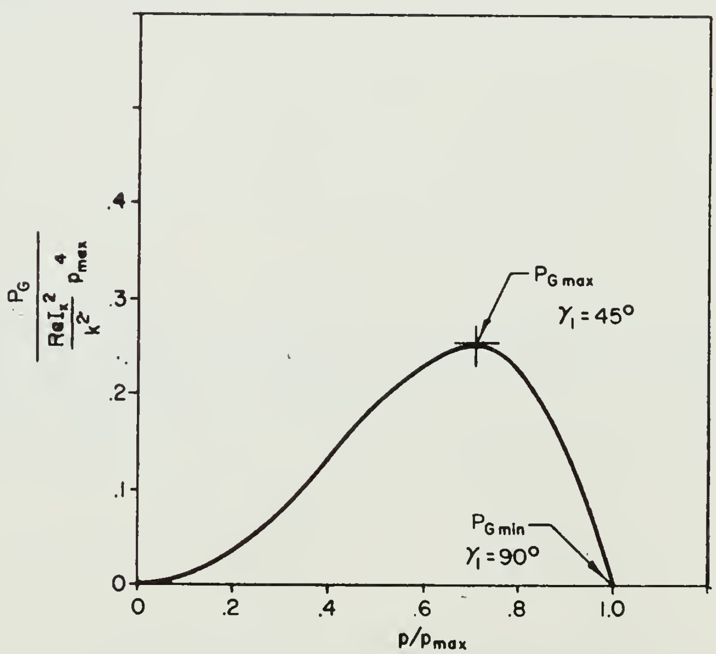


Figure 5.31.1 Plot of Gyro Torque Motor Power for Steady Roll Rate in Non-Dimensional Parameters.

The preceding Figure illustrates a maximum point which occurs at a gimbal angle of 45 degrees as well as a minimum point occurring at a gimbal angle of 90 degrees.

To determine the transient energy required to drive the gimbal angle to 90 degrees it is necessary to assume a roll acceleration. In the practical case the vehicle is torqued by the control system within the allowable strength of the supporting structure of the control system. Thus, let the spacecraft be torqued so that this maximum angular acceleration is held until the vehicle reaches the desired steady roll rate. If the allowable roll acceleration is $\dot{p}_{max}$ and if the inertias of the case and gimbals are neglected the control power is given by equation 5. 31. 9 where $p = \dot{p}_{max}\, t$. Integrating this to a value of time equal to $p_{max}/\dot{p}_{max}$ gives.

$$E_{G_{Transient}} = \left(\frac{R_a I_x^2 p_{max}^5}{k^2 \dot{p}_{max}}\right)\left(\frac{1}{3}\left\{\frac{p}{p_{max}}\right\}^3 - \frac{1}{5}\left\{\frac{p}{p_{max}}\right\}^5\right)$$

(Eq. 5. 31. 10)

If t_i represents the increment of time consumed in the transient then the last equation can be written in another form.

$$E_{G_{Transient}} = \left(\frac{R_a I_x^2 p_{max}^3}{k^2}\right)\left(t_i\, p_{max}\right)\left(\frac{1}{3}\left\{\frac{p}{p_{max}}\right\}^2 - \frac{1}{5}\left\{\frac{p}{p_{max}}\right\}^4\right)$$

(Eq. 5. 31. 11)

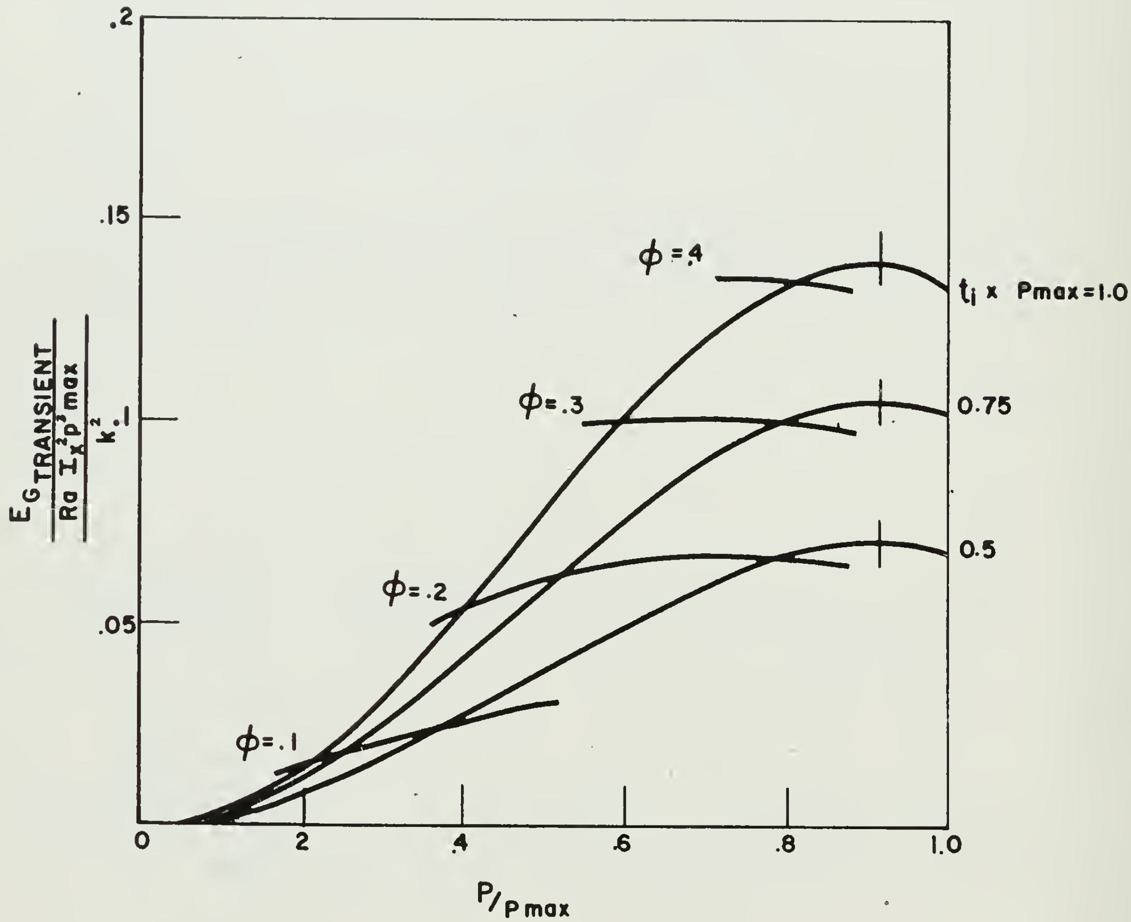


Figure 5. 31. 2 Plot of Torque Motor Energy Required to Torque a Pair of Gyro Controllers from a Zero Position to a Steady State Position. See Equation 5. 31. 11.

The angle ϕ represents the attitude change during the transient.

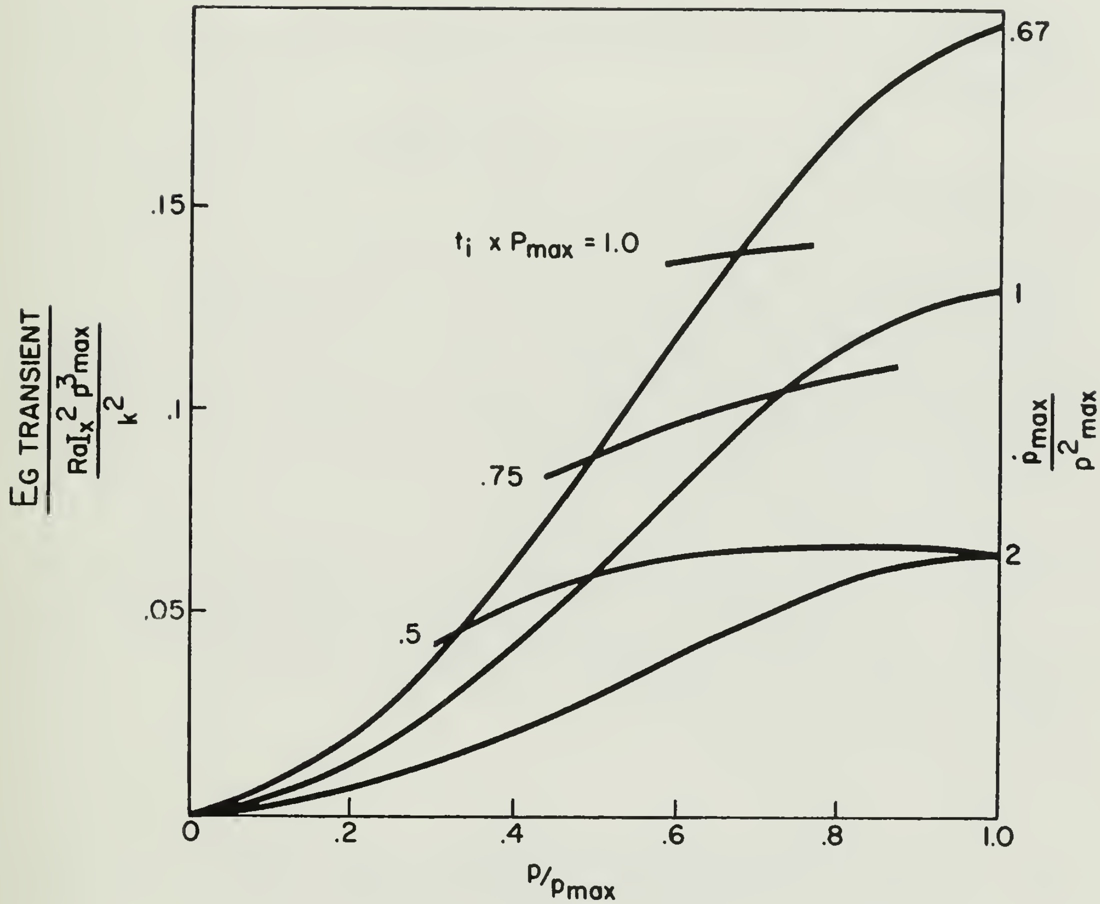


Figure 5. 31. 3 Plot of Torque Motor Energy Required to Torque a Pair of Gyro Controllers from a Zero Position to a Steady State Position. See Equation 5. 31. 10.

The plots of Figures 5. 31. 2 and 5. 31. 3 are the same equation with different parameters for the constant contours. In Figure 5. 31. 2 the primary curves are for constant non-dimentional time with angular displacement cross-plotted. In Figure 5. 31. 3 the lines of constant contours are $\frac{\dot{p}_{max}}{p^2_{max}}$ which is interpreted as the non-dimensional angular acceleration variable, $\dot{p}_{max}$.

The total energy required to make an attitude change will consist of the transient energy plus the steady state power integrated over the time period of the maneuver. No energy is required for the recovery to a steady non-rolling attitude, but it is assumed that the time required in the recovery is equal to the time consumed in the initial transient. Let the total attitude change be ϕ which is related to the other parameters as follows.

$$\phi = p_{s.s.} \left(t_T - t_i \right) \qquad \text{(Eq. 5. 31. 12)}$$

where t_T is the total time of the maneuver.

This gives for the total energy,

$$E_{G\phi} = \frac{R_a I_x^2}{k^2 \dot{p}_{max}} \frac{p^5_{max}}{} \left[\frac{1}{3} \left\{ \frac{p_{s.s.}}{p_{max}} \right\}^3 - \frac{1}{5} \left\{ \frac{p_{s.s.}}{p_{max}} \right\}^5 \right] + \int_{t_i}^{t_T - t_i} P_G \, dt$$

(Eq. 5. 31. 13)

which integrates to the following.

$$E_{G\phi} = \frac{R_a I_x^2}{k^2 \dot{p}_{max}} \frac{p_{max}^5}{} \left\{ \frac{4}{5} \left(\frac{p_{s.s}}{p_{max}} \right)^5 - \left[\frac{2}{3} + \frac{\phi \dot{p}_{max}}{p_{max}^2} \right] \left(\frac{p_{s.s}}{p_{max}^2} \right)^3 + \frac{\phi \dot{p}_{max}}{p_{s.s}^2} \left(\frac{p_{s.s}}{p_{max}} \right) \right\}$$

(Eq. 5. 31.13a)

Equation 5. 31.13a is difficult to analyze for minima and maxima, and it is necessary to make an approximation. Examination of the energy required in the transient equation 5. 31. 10 for realistic values of the parameters shows that the transient energy is small if the time of the transient is small, and therefore can be neglected except for the region at large p/p_{max} where the steady state power is small. Using this approximation gives an equation for the control energy as follows.

$$E_{G\phi} = \left(\frac{R_a I_x^2}{k^2} p_{max}^3 \right) \phi \left(\frac{p_{s.s}}{p_{max}} - \left\{ \frac{p_{s.s}}{p_{max}} \right\}^3 \right)$$

(Eq. 5. 31. 14)

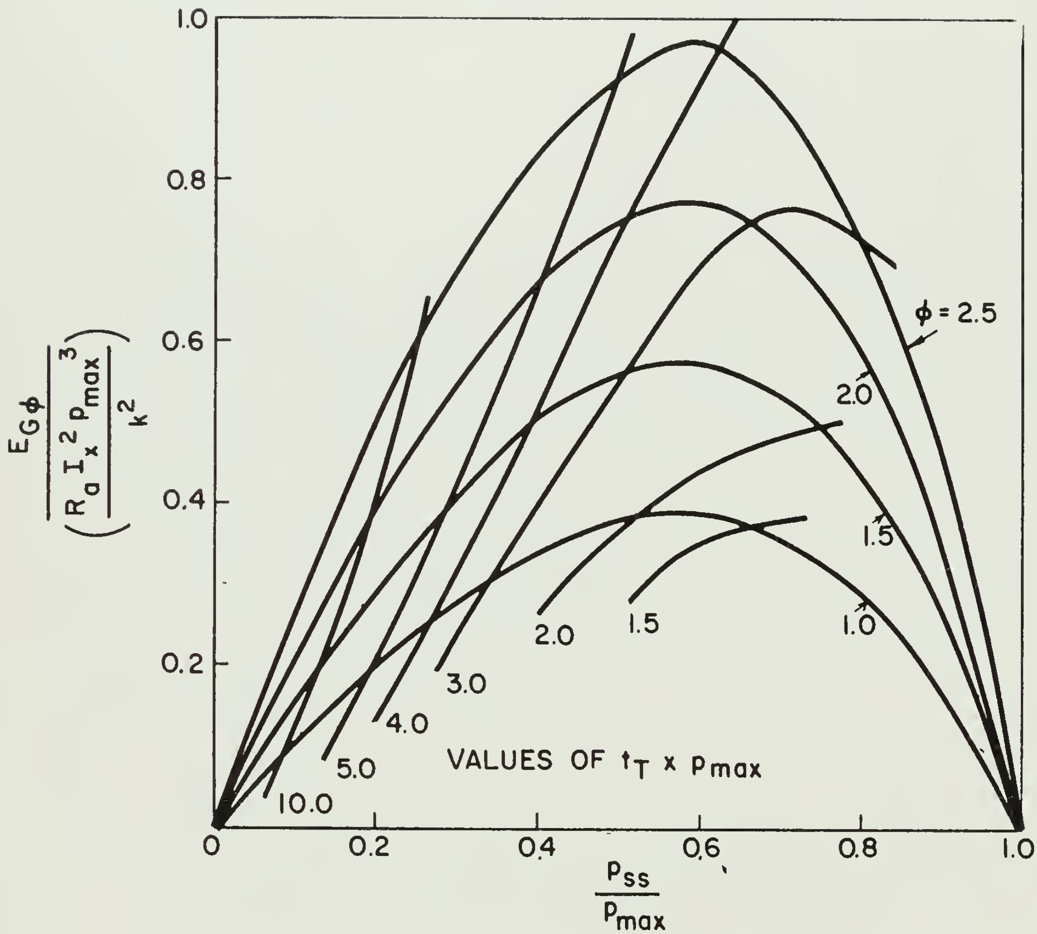


Figure 5.31.4 Energy Required by the Torque Motor of a Pair of Gyro Controllers to Execute a Steady Roll of Displacement, ϕ. Cross-plotted are the non-dimentional Times Required in the Maneuver. See Equation 5.31.14.

Using the equations for the wheel and gyro energies the two systems can be compared in a number of ways. First consider the energy required by the inertia reaction wheel system to accelerate the vehicle to p_{max} as compared

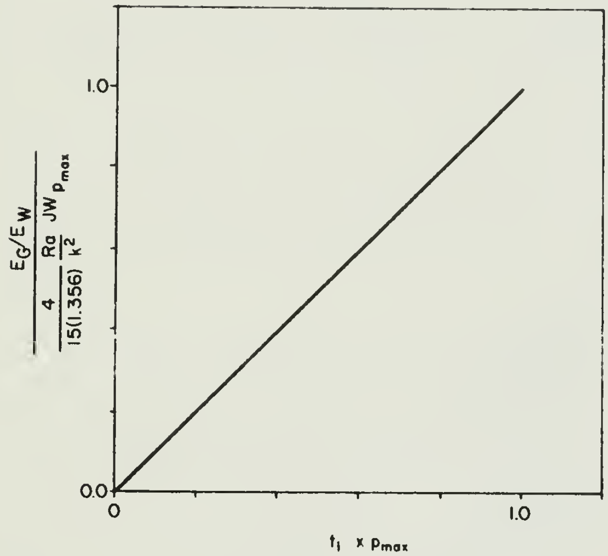


Figure 5.31.5 Comparison of energies of wheel system to twin gyro system to accelerate the spacecraft to p_{max}. See Equation 5.31.15

with that of the gyro system. The ratio of the two energies are as follows.

$$\frac{E_G}{E_W} = \left(\frac{4}{15\,(1.356)}\ \frac{R_a}{k^2}\ J_W\ p_{max}\right)\left(t_i\ p_{max}\right)$$

(Eq. 5.31.15)

This relation is plotted in Figure 5.31.5 and is favorable to the gyro system provided the transient times are not excessively long.

Another comparison of the wheel to the gyro can be made

on the energy required in a steady roll. The wheel system must only get the vehicle moving at a steady roll rate if we neglect rotational losses of the wheel whereas the gyro must exert a steady torque during the time the vehicle is in the maneuver. The comparison gives the following ratio.

$$\frac{E_G}{E_w} = \left(\frac{2\, R_a\, J_w}{1.356\, k^2}\; p_{max} \right) \left(t_T\, p_{max} \right) \left(1 - \left\{ \frac{p}{p_{max}} \right\}^2 \right)$$

(Eq. 5. 31. 16)

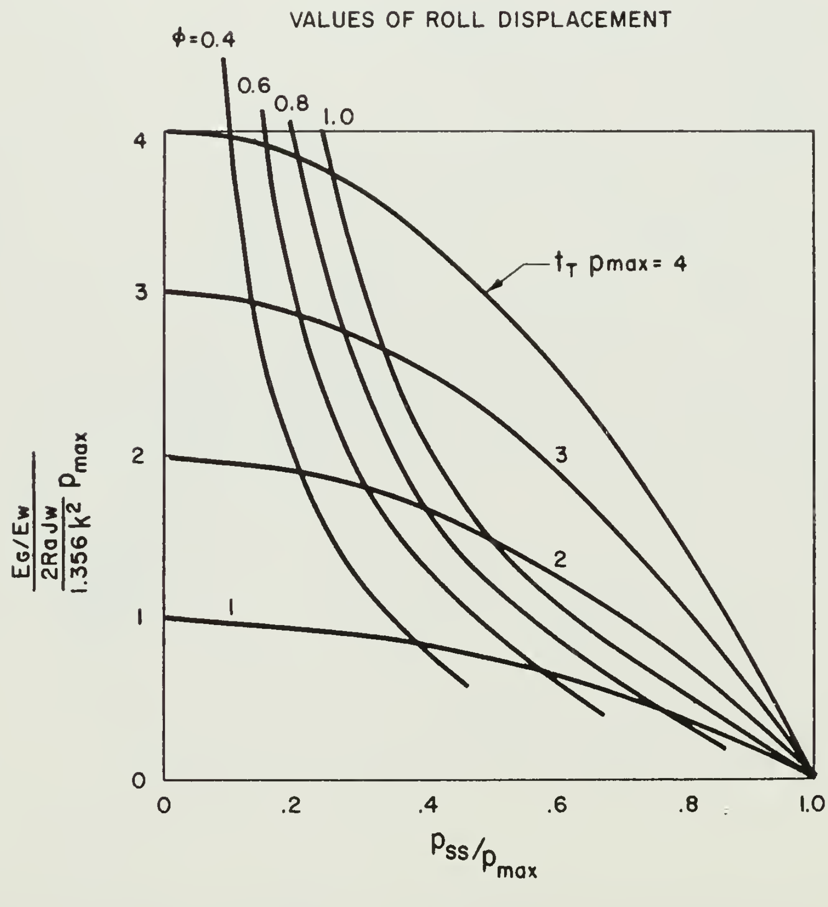


Figure 5. 31. 6 Comparison of energies of a Wheel System to a Twin Gyro System in a Steady Roll. Curves are plotted for various total times with roll displacement cross-plotted. See Equation 5. 31. 16.

Figure 5. 31. 6 shows that, in general, the gyro is more favorable at the higher roll rates. For an armature resistance of 4 ohms, a torque motor sensitivity of 6. 2 lbs ft/amp, a wheel inertia of 400 lbs ft sec^2, and a p_{max} of 0. 1 degree per second the normalizing factor in Figure 5. 31. 6 is 0. 107 which makes the ratio E_G/E_w less than about 0. 4 for all points on the Figure, but if either the wheel inertia or the p_{max} is increased by a factor of ten, then the energy ratio for a given roll displacement is increased by a factor of ten which means that the gyro control energy is four times the wheel energy for the long, slow rolling maneuver. In this case it is more economical for the gyro to operate at higher gimbal angles and drive the spacecraft at faster rates.

A third comparison of the control energy of a gyro controller compared with a wheel can be made by assuming a motor with level torque-speed characteristics over a range. Granted that this motor may not be an efficient one, but the device does permit a direct comparison of two systems using the same driving motor. Previous comparisons have assumed an ideal motor to drive the wheel whereas actual motor characteristics were used for the gyro torque motor. Consider then a motor whose power is directly proportional to the torque delivered to the load. Thus let the motor be characterized by the equation.

$$P = k M \qquad \text{(Eq. 5. 31. 17)}$$

For the wheel system the energy delivered to the motor is found to be as follows:

$$E_w = k I_x p \qquad \text{(Eq. 5. 31. 18)}$$

For the gyro system using equation 5. 31. 5 the energy required by the gyro torque motor is determined to be as follows:

$$E_G = k \, 2H \, p (\cos \gamma_1) \, t \qquad \text{(Eq. 5. 31. 19)}$$

Dividing the last two equations and making use of equation 5.31.6 one arrives at the following ratio:

$$E_G/E_w = \phi \cot \gamma_1 \qquad \text{(Eq. 5.31.20)}$$

This ratio plotted in Figure 5.31.7 further indicates that the gyro must operate at large gimbal angles to be competitive with the wheel for large displacements.

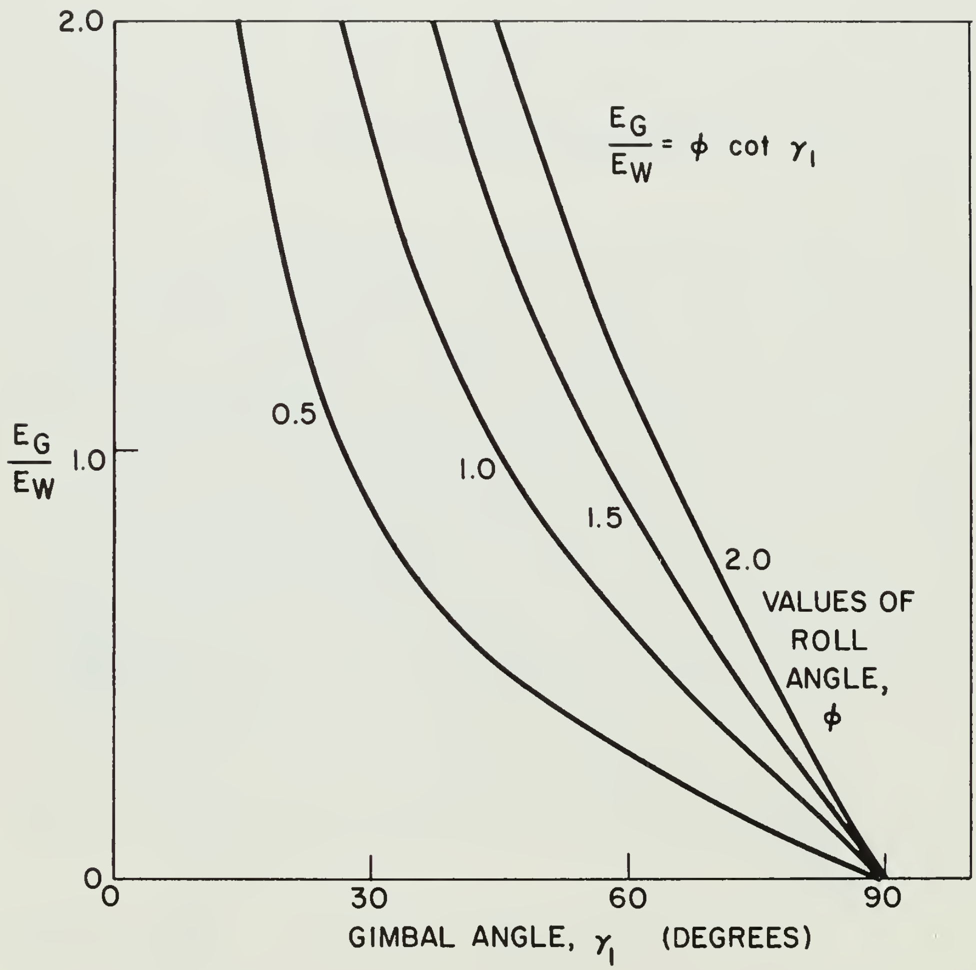


Figure 5.31.7 Comparison of Gyro Control Energy to Inertia Reaction Wheel Control Energy based on Both Systems being Driven by the same Motor whose Characteristics are that the Power Requires is Directly Proportional to the Torque Delivered to the Load.

To conclude the comparison of the inertia reaction wheel to the gyro it may be said that for equivalent systems the gyro controller consistently requires less control power than the wheel. However, for large attitude changes in which the gyro torque motor must exert a steady torque during the turn whereas the wheel motor is free-wheeling, the ratio of gyro torque motor energy to wheel torque motor energy for a given attitude change is progressively reduced as the angular rate is increased. This trend is shown in Figure 5. 31. 6, and there is indicated that a minimum in the ratio exists when the rate variable is driven to its maximum. This position for the gyro controller represents a gimbal angle of 90 degrees at a point where all of the angular momentum of the controller has been transferred to the vehicle. At a gimbal angle of 90 degrees the component of the gyro controller angular momentum vector normal to the attitude rate vector vanishes, and hence the control moment is zero. For an attitude change at this rate the gyro control power is simply the transient power shown in Figures 5. 31. 2 and 5. 31. 3. Stated in different words, in the operation of transferring angular momentum to the vehicle, the wheel must be given a substantial change in kinetic energy whereas the gyro controller does not require an overall energy change to exchange momentum with the vehicle. Therefore, it is not surprising to find that the gyro controller requires less control power to execute a given maneuver.

5.32 Comparison of Inertia Wheel Controller to Gyro Controller for a Statistical Disturbance

A comparison of the inertia reaction wheel to the gyro controller on a steady state statistical basis is complicated by the fact that a well-behaved torque disturbance when applied to the spacecraft which acts like an ideal integrator does not yield a finite steady state mean square value for the rate variables. BATTIN[(50)] discusses this result and shows that the value grows without limit and does not approach a stationary random process. Thus for the gyro if we assume small values of the rate variable, p, the control power given by equation 5.31.9 is

$$P_G = \frac{R_a I_x^2}{k^2} \; p^2 \max p^2 \qquad \text{(Eq 5.32.1)}$$

Thus the mean control power is given by

$$P_{G\,(average)} = \frac{R_a I_x^2}{k^2} \; p^2 \max \overline{p^2} \qquad \text{(Eq 5.32.2)}$$

but $\overline{p^2}$ for the disturbance power density spectrum of section 4.42 is infinite although the torque disturbance has a zero mean value.

On the other hand, the average power for the reaction wheel found by differentiating with respect to time equation 5.31.4 and taking the average as follows.

$$P_{W\,(average)} = \frac{I_x^2}{J_W} \; \overline{p \, \dot{p}} \qquad \text{(Eq 5.32.3)}$$

However, it is easily shown by finding the cross-power density spectrum of p and $\dot{p}$ that $\overline{p \, \dot{p}}$ identically vanishes simply because the cross-power density spectrum is an odd function. This result suggests that a torque motor device capable of providing positive energy as well as storing energy provided to the device by the

wheel could operate with no net input energy provided the wheel has a sufficient quantity of initial energy to get the process started. Because of the difficulties with the statistical analysis using the vehicle rate variables, the comparison must be made on a different basis. Accordingly consider the assumption that if the control system performs its function well that all of the disturbances will be isolated from the vehicle. If it is further assumed that the wheel has no back emf for small speeds and that the gyro gimbal viscous friction is large compared to the gimbal inertia the following comparison can be made.

For the wheel, the torque resulting from a control voltage is given by

$$T_W = V_M K_{TW} \qquad \text{(Eq 5.32.4)}$$

This gives for the power density spectrum of the control voltage for the wheel the following.

$$\Phi_{WW}(\omega) = \frac{1}{K_{TW}^2} \Phi_{TT}(\omega) \qquad \text{(Eq 5.32.5)}$$

For the gyro, if k is the viscous friction coefficient of the gimbal

$$T_G = \frac{K_{TG}\, 2H}{k} V_M \qquad \text{(Eq 5.32.6)}$$

This gives for the power density spectrum of the control voltage for the gyro the following.

$$\Phi_{GG}(\omega) = \frac{k^2}{K_{TG}^2\, 4H^2} \Phi_{TT}(\omega) \qquad \text{(Eq 5.32.7)}$$

The ratio is given by

$$\frac{\Phi_{GG}(\omega)}{\Phi_{WW}(\omega)} = \left(\frac{k}{2H}\right)^2 \left(\frac{K_{TW}}{K_{TG}}\right)^2 \qquad \text{(Eq 5.32.8)}$$

The factor $\left(\frac{k}{2H}\right)$ can be made very small, and the characteristics of torque motors for wheel systems and gyro systems follow the trends of Figure 5.32.1; therefore, the factor

$$\left(\frac{K_{TW}}{K_{TG}}\right)$$

is also a small quantity leading to the conclusion that the power density spectrum for the gyro control voltage is exceedingly small as compared to the wheel system.

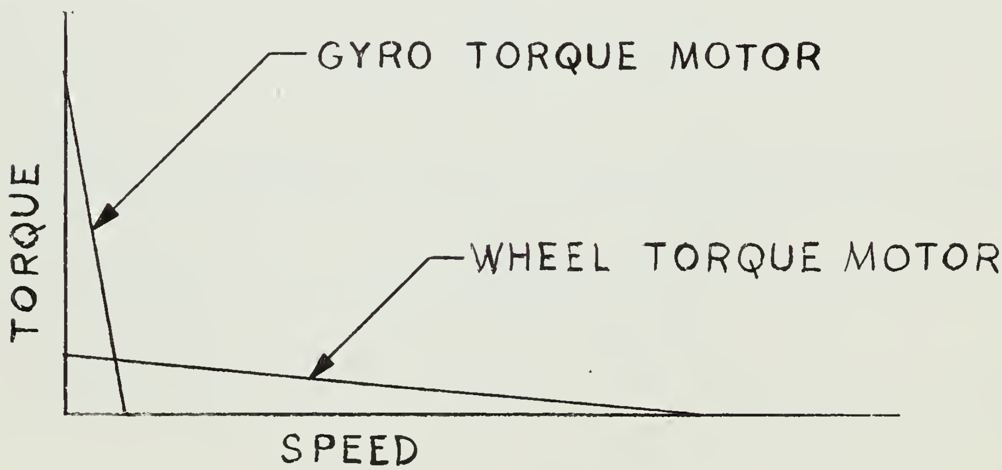


Figure 5.32.1 Typical torque-speed curves for motors used in inertia reaction systems and gyro systems.

5.33 Considerations of Mass Expulsion Attitude Control Systems

Because of the fundamental differences between mass expulsion systems and momentum exchange systems it is difficult to find a criterion other than overall system weight. Accordingly let us examine the fuel requirements for a mass expulsion system in an attempt to get some idea of the system weight of this system. It is assumed that the spacecraft attitude is to be controlled at all times which indicates that a discontinuous-pulsed type mass expulsion system operating in a limit cycle is suggested. Continuous systems with linear valve characteristics tend to be wasteful of fuel compared to the limit cycle type operation. The limit cycle is defined by the excursions of the vehicle rate and attitude variables shown in Figure 5.33.

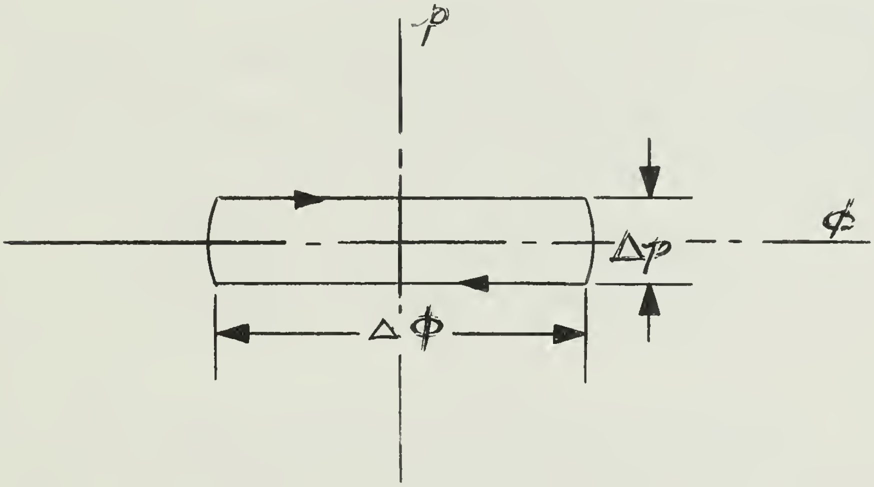


Figure 5.33 Limit Cycle of Mass Expulsion System

The period of the limit cycle is seen to be equal to the following.

$$P_{LC} = \frac{4\Delta\phi}{\Delta p} \qquad \text{(Eq. 5.33.1)}$$

Solving for Δp gives

$$\Delta p = \frac{4 \Delta \phi}{P_{LC}} \qquad \text{(Eq. 5.33.2)}$$

The total torque impulse per cycle is given by

$$(M \Delta t) = I_x \, 2 \Delta p = \frac{8 \, I_x \, \Delta \phi}{P_{LC}} \qquad \text{(Eq. 5.33.3)}$$

On an average basis the torque impulse per unit time is given by the following.

$$\text{Torque Impulse/Unit Time} = \frac{8 I_x \Delta \phi}{P_{LC}^{2}} \qquad \text{(Eq. 5.33.4)}$$

The mass of the fuel is related to the thrust impulse by its specific impulse, I_{sp}, which for a lever arm of 1 from the center of mass of the spacecraft gives the following for the pounds of fuel mass consumed per unit time.

$$\text{LBS Fuel Used/Unit Time} = \frac{8 I_x \Delta \phi}{I_{sp} \, 1 \, P_{LC}^{2}}$$

(Eq. 5.33.5)

In this last equation I_x represents the moment of inertia of the spacecraft and will vary for each phase of the mission. Therefore, the total fuel consumed for attitude control purposes can be written as this equation.

$$\text{Total Fuel} = \frac{8 \Delta \phi}{I_{sp} \, 1 \, P_{LC}^{2}} \sum_{i=1}^{N} I_{xi} \, t_i$$

(Eq. 5.33.6)

Consider an example with the following numerical data which may apply for a 400 day Mars excursion.

$$\phi = 2 \text{ degrees}$$

$$I_{sp} = 320 \text{ seconds}$$

$$l = 20 \text{ feet}$$

$$\sum_{i=1}^{N} I_{xi}\, t_i = 1.0 \times 10^{13} \text{ lb-ft-sec}^3$$ (Based on 200 days each in Phase D and H.)

$$\text{Total Fuel} = \frac{4.37 \times 10^9}{P_{LC}^{\;2}} \qquad \text{(Eq. 5.33.7)}$$

with P_{LC} expressed in seconds

Equation 5.33.7 plots as an inverse square function and several values for particular limit cycles are as follows:

P_{LC}	Fuel Weight
1 min	12,100 pounds
5	4,850
10	1,213
15	540
30	135
60	33

This example was made for the roll fuel which is small compared with the fuel required for pitch and yaw, each of which is sixteen times greater than the requirement for roll control. Attitude fuel for continuous control is approximately thirty two times the above fuel weights which indicates that unless very long limit cycles are provided continuous control by mass expulsion will not be practical.

One solution to the problem which appears to offer considerable benefit to the astronauts as well as reduce the attitude control fuel consumption is to spin stabilize the spacecraft. The artificial gravity provided is definitely beneficial to the crew. Ehricke[51] has completed a detailed study of a Mars trip, and he proposes to spin stabilize the vehicle to the order of providing the spacecraft commander about 0.25 times the acceleration of gravity (g). The total control fuel required for spin-up is 6900 pounds for one model of the spacecraft and 4232 pounds for another. If the crew moves about in a spin stabilized vehicle there results a nutation of the motion of the spacecraft which probably will be required to be damped. A momentum exchange system, particularly a gyro type system, could efficiently damp undesirable motions of a spin stabilized spacecraft, but they cannot substantially control the attitude of the spacecraft.

It can be concluded that momentum exchange systems certainly cannot replace the mass expulsion systems because their momentum storage capability is limited and they tend to become saturated when the vehicle is subjected to torques with a non-zero mean value; however since most of the disturbance torques either have a zero mean value, or the spacecraft can be operated such that the mean value of the torque is a small number, the momentum exchange system can augment the mass expulsion system to provide continuous control without ejection of fuel mass. The physical quantity consumed by the momentum exchange system is energy which can be generated on a scheduled basis or can be supplied from the energy of the sun. Clearly, any mass that is expelled from the vehicle is non-retrievable, and consequently any mass expulsion system must carry an adequate supply of control fuel plus a reserve quantity, and yet the astronauts are still faced with the threat of loss of control in the event all fuel is consumed. On this basis it seems reasonable to proceed in choosing a momentum

exchange system separate from the provision of a mass expulsion system.

5.4 Peak Power

In the usual operation the gyro controller consumes power to spin the rotor, and it comsumes control power to torque the spin axis. Let us compare the peak control power of the gyro torque motor to the peak power delivered to the wheel by the wheel motor, again favoring the wheel by choosing the point of comparison at the peak power point of the gyro shown in Figure 5.31.1. At a roll rate of $p = p_{max} / \sqrt{2}$ the wheel power is given by the equation, where $\dot{p}$ is held constant at $\dot{p}_{max}$

$$P_W = \frac{I_x^2}{J_W} p\dot{p} = \frac{I_x^2 \; p_{max}^2}{2 J_W t_p} \qquad \text{(Eq. 5.4.1)}$$

where $t_p = p/\dot{p}_{max}$ the rise time.

The corresponding equation for the gyro power is given by

$$P_G = \frac{R_a I_x^2}{4 k^2} p_{max}^4 \qquad \text{(Eq. 5.4.2)}$$

Dividing the two gives the ratio

$$\frac{P_G}{P_W} = \frac{R_a J_W p_{max}^2}{2 (1.356) k^2} \times t_p \qquad \text{(Eq. 5.4.3)}$$

This curve is a similiar one as that plotted in Figure 5.31.5 except for a different normalizing factor and here t_p represents the time to achieve a roll of $p_{max} / \sqrt{2}$ from an initial rest position. Using the numerical data following Figure 5.31.6 together with a time, t_p, of ten seconds gives the ratio P_G/P_W peak of 0.000462 showing the peak power of the gyro control torque motor is just a fraction of the peak power required for the inertia wheel control motor. Another approach even more favorable to the wheel is to allow the wheel to accelerate at constant power. In this case $p\dot{p}$ = constant and can be integrated.

The result is to remove the factor of 2 in equation 5.4.3 and hence the example yields 0.000924, a value that is twice the ratio found initially to be 0.000462.

5.5 System Weight

The system weight is approximately proportional to the number of wheels or gyro controllers in the system. It appears that a single gyro rotor for the Mars excursion will be on the order of 500 pounds. To support this mass requires an additional 300 pounds of structure. The torque motors will have a mass of about 100 pounds and the associated electronics control mechanisms add another 100 pounds. Thus a single controller will have a combined mass of about 1000 pounds. An inertia wheel controller will require a larger mass in the rotor and a larger torque motor but the structure supporting the wheel will be less. Therefore, a wheel system will be of the same order of magnitude in mass as the gyro system. A single degree of freedom controller may be as much as 200 pounds lighter than the two-degree-of-freedom controller because it requires only a single torque motor and does not require the additional structure to give the extra degree of freedom. Therefore the following weight summary is considered realistic.

3 controllers	(wheel)	3000 pounds
3 controllers	(SDF)	2400 pounds
4 controllers	(TDF)	4000 pounds
6 controllers	(SDF)	4800 pounds

5.6 Summary

Since the torque disturbances acting on a spacecraft either have a zero mean or the mean torque can be made small by proper design and operation of the spacecraft, it is feasible to control the attitude of the vehicle by momentum exchange methods for extended periods of time without expenditure of fuel mass. Attitude control by pure mass expulsion systems operating in a limit cycle promises to consume substantial quantities of fuel if continuous control is provided. Accordingly, to minimize the expenditure of fuel it seems reasonable to employ the momentum exchange system on a continuous basis to provide the precise attitude control required and use the mass expulsion system intermittently to desaturate the momentum exchange system. Because of the possibility of saturation of the momentum exchange system the vehicle cannot be without a mass expulsion system, and because of the possibility of control fuel exhaustion, the momentum exchange system is justified, and it appears that neither can replace the other, and both are needed. Accordingly, the momentum exchange system is assumed to be required and will be selected without regard to the mass expulsion system required for the mission.

In the matter of the choice between inertia reaction wheel versus gyro controllers the over-all power requirement will probably average to be about the same; however, the gyro controller control power is a small fraction of the wheel control power, and this is particularly true for stabilizing the vehicle in an inertially fixed attitude. Power can be provided to the gyro controller at a constant rate whereas the wheel system demands power at a varying rate with high peak power necessitating design to the peak rating for all of the associated control components. In the usual selection of a control system the final factor is "designers choice", and although response characteristics were not considered in the initial choice of a control system in this chapter, they should be. The gyro system as shown in

subsequent chapters provides passive stabilization not provided by inertia reaction wheels. The above factors, although not overwhelming, favor the gyro controller.

The four controller system shown in Figure 3. 3 is considered to be superior to the other systems considered for the following reasons:

1. The overall realiability of the system is higher because of its inherent adaptive features.

2. The average power consumption for the several modes of pointing is equal to or less than the other systems.

3. The ejection of mass from the vehicle can be made equal to or less than that required for the other systems.

4. The overall system weight is considered to be competitive since any three gyro system would probably have to carry a spare controller to achieve reliability requirements.

Subsequent chapters will consider only the four gyro system.

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